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Dark Spatial Solitary Wave Due to Cascaded $\chi^{(2)}$ Nonlinearity*

GAO Guan-Qie, CHEN Xian-Feng,† CHEN Yu-Ping, WANG Fei-Yu, and XIA Yu-Xing

Institute of Optics and Photonics, Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China

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Abstract *The formation of the dark spatial solitary wave in cascaded second harmonic generation processes is numerically studied based on the nonlinear-coupled equations. It is shown that the solitary wave exists when the effective three-order nonlinearity induced by cascaded second-order nonlinearity is negative.*

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Key words: cascaded nonlinearity, dark spatial soliton

1 Introduction

It has long been known that the cascaded second-order ($\chi^{(2)}:\chi^{(2)}$) nonlinearity can lead to effective third-order nonlinearity,[1] and this has been widely appreciated since the series of experiments were reported and were explained by the theoretical predictions.[2−4] Optical waves propagating in materials with substantial dispersion or diffraction and significant χ^3 nonlinearity can be described by the nonlinear Schrödinger equation (NLSE). Spatial soliton or solitary wave corresponds to the balance between diffraction and nonlinear effect during the propagation. Cascading of second-order nonlinearities ($\chi^{(2)}:\chi^{(2)}$) in quadratic media, which is identified as an approach for generating large, intensity-dependent phase shifts, can also in principle create spatial solitary wave. Involving quadratic nonlinearity ($\chi^{(2)}$), a formation of one- and two-dimensional bright solitary waves was proposed theoretically in 1976.[5] In recent years much attention has been paid to the underlying physics of the bright soliton.[6−9] Further more, the spatial[10] and temporal[11] optical solitons resulting from multistep cascading have been studied, which can obviously reduce the input intensity required for the formation of the solitons. The physics of the dark soliton due to cascaded second-order nonlinearity at the stable state was theoretically studied.[12]

In this paper, we numerically investigate the beam with dark profile in Gaussian background evolving process in second-order nonlinearity media when phase-mismatch is negative. We succeeded in finding the formation of dark solitary wave due to the cascaded second-order nonlinearity.

2 Spatial Soliton in Cascaded Processes

To simply explain the formation of the spatial soliton, we can start from Ref. [3].

Positive or negative mismatch during the cascaded $\chi^{(2)}$ processes can induce positive or negative effective nonlinear refractive index, respectively (the mismatch is defined conversely in Ref. [3]). In the bright soliton situation, the positive effective nonlinear refractive index due to positive mismatch provides an additional refractive index distribution, which is proportional to the intensity distribution. Because the refractive index of the middle part is larger than that of the sides, the light beam is trapped. While for the dark soliton, the negative mismatch is required. Due to the dark soliton intensity distribution, the effect refractive index of the middle part is also larger than that of the sides, so the light beam also converges, which may be balanced by diffraction.

For further consideration, in the slowly varying envelope approximation, continuous-wave (cw) light beams evolution in a medium with a large quadratic nonlinearity under type-I condition can be described by the wave coupling equations. They used to be reduced in normalized form[9]

$$\mathrm{i}\,\frac{\partial a_1}{\partial\xi}+\frac{1}{2}\nabla_\perp^2 a_1+d(\xi)a_1^* a_2\exp(-\mathrm{i}\,\beta\xi)=0,$$
$$\mathrm{i}\,\frac{\partial a_2}{\partial\xi}+\frac{\alpha}{2}\nabla_\perp^2 a_2-\mathrm{i}\vec{\delta}\delta\cdot\nabla_\perp a_2+d(\xi)a_1^2\exp(\mathrm{i}\,\beta\xi)=0\,, \quad (1)$$

where a_1, a_2 are the normalized amplitudes of the fundamental and harmonic waves respectively, $\alpha=k_1/k_2$, k_1, k_2 are the wave numbers at the two frequencies. $d(\xi)$ is the normalized quadratic nonlinearity and $\beta=k_1\eta^2\Delta k$, where $\Delta k=2k_1-k_2$ is the wave-vector-mismatch and η is the characteristic beam transverse width. ξ is the propagation distance in the unit of $k_1\eta^2$. δ denotes the Poynting vector walk-off when the propagation direction is not along the crystal optical axes. We can set $\delta=0$, when there is no walk-off between the fundamental and the harmonic waves (for example: in typical QPM geometries). $\nabla_\perp$ equals $\partial/\partial s$ in the situation of one dimension,

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†Correspondence author, E-mail: xfchen@sjtu.edu.cn

here s is the normalized transverse coordinate in unit of η.

3 Numerical Investigation

In our simulation, a Gaussian beam with a thin hollow introduced at the center, described as $a_1(s) = A_0 \exp(-s^2/\sigma^2)[1-\mathrm{sech}^2(s)]^{1/2}\,\mathrm{sign}\,(s)$, is injected at the entrance. Here σ is the width of the background Gaussian beam. We set $\sigma = 20\eta$ and then the width of the hollow is about 1.70η. To investigate the beam evolution, we integrate Eq. (1) numerically using a split-step approach. The linear part ($\nabla_\perp^2$) is integrated in the Fourier space and the nonlinear part is integrated by a fourth Runge–Kutta algorithm. We divide the evolving process along the propagation direction into many steps. In every step, the nonlinear effect is firstly considered exclusively, and then we only consider diffraction processes. In our simulations, the input beams do not coincide with the stationary solution given in Ref. [12].

We investigate the fundamental wave intensity distribution at the output section after about 25 diffraction lengths ($k_1\eta^2$). As same as the temporal dark soliton,[13] the spatial dark soliton due to cascaded quadratic nonlinearity is also topological. At the entrance we must introduce a π phase step between the two sides of the hollow as the inset of Fig. 1, otherwise the hollow at the center will split into two ones during the evolving process.

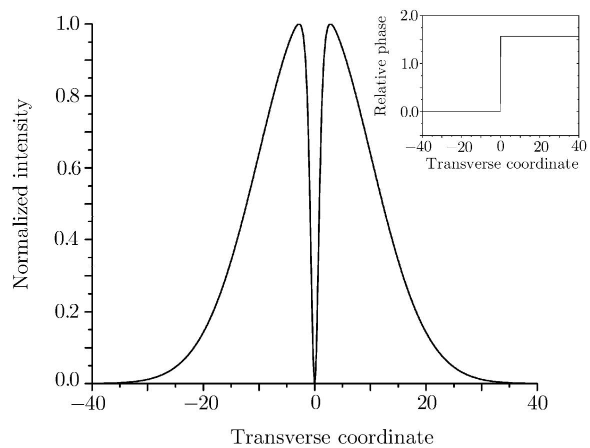

Fig. 1 Intensity and phase distribution along the transverse coordinate of the input fundamental wave. There is π step between the two sides of the hollow.

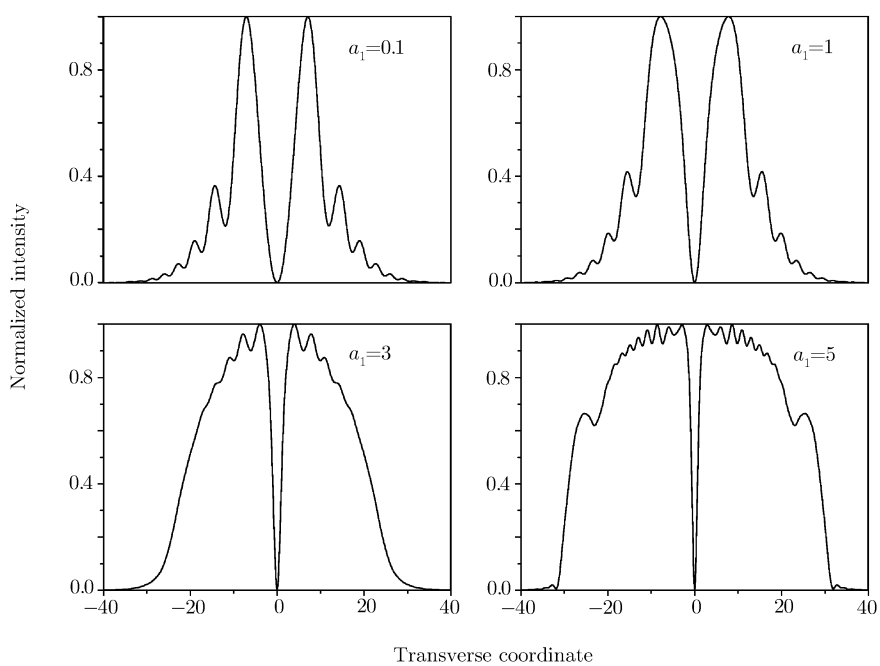

Fig. 2 The fundamental wave intensity distribution at the output section with different input fundamental intensities. The amplitude of the fundamental is 0.1, 1, 3, and 5 respectively.

We set wave vector mismatch as $\beta = -15$, and normalized amplitude as $a_1 = 0.1, 1, 3$, and 5, respectively. Figure 2 shows the normalized intensity distribution of fundamental wave at the output section after the beam propagates about 25 diffraction lengths. As we can see in Fig. 2, when the initial intensity of the fundamental wave is small, the central hollow expands. The effect of diffraction dominates when the intensity is low enough.

As the intensity a_1 increases, the width becomes smaller and smaller. That means self-focusing effect induced by cascaded second order optical nonlinearity plays an important role. When the amplitude reaches to $a_1 = 5$, the width is calculated to be 1.54 η, which is a little smaller than the input width. It is evident that the solitary-like wave forms when the input intensity of fundamental wave is high enough. At the same time, the transverse width of the Gaussian beam becomes wider because of the diffraction and defocusing effect of the bright Gaussian background.

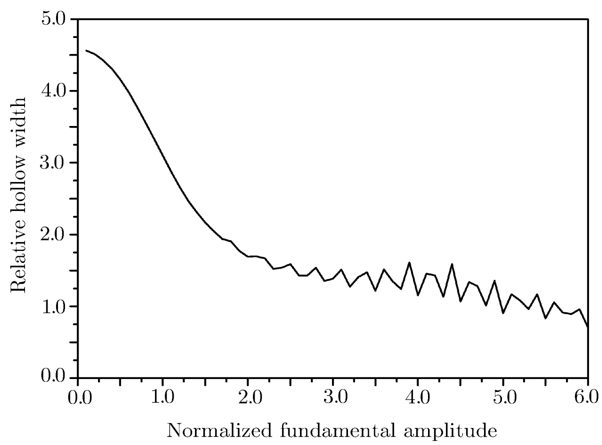

Fig. 3 The relative hollow width as a function of the input fundamental wave amplitude. When the amplitude of the fundamental is big enough, the width of the hollow will become fixed, or even smaller.

Figure 3 depicts the relative hollow width varying with the input fundamental amplitude. The relative hollow width decreases very fast when the amplitude increases before $a_1 = 2$. After $a_1 = 2$, the relative width becomes almost fixed. As seen in the figure, there is small modulation of the relative width. We believe that it is possible due to the calculation accuracy. In our simulation, the total intensity of the fundamental wave does not change much, and only a little energy, about 5 percent, flows into the harmonic wave.

To experimentally generate the dark solitary wave, we need to introduce a hollow at the center of a Gaussian beam, and we should add a relative π phase distortion at one side of the hollow. For the crystal of periodically poled lithium niobate, the normalized intensity of the fundamental wave corresponds to the peak intensity of $0.1 \sim 1$ GW/cm^2, which is available by using pulsed solid lasers. For $\eta = 10$ μm, twenty diffraction length is about 25 mm, which is a usual length for periodically poled lithium niobate crystal.

4 Conclusion

In conclusion, we have investigated the formation of the dark spatial soliton by cascaded second-order nonlinearity with negative wave vector mismatch. By using a split-step method, we found that a power threshold ($a_1 = 2$) is required for spatial dark soliton formation under negative phase-mismatch condition.

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