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# Competition of frequency conversion and polarization coupling in periodically poled lithium niobate

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**ABSTRACT** In this paper wave-coupling equations including polarization coupling and frequency doubling in periodically poled lithium niobate (PPLN) is proposed. Numerical solutions of the coupling equations show that there is a competition between the polarization coupling and second-harmonic generation in a PPLN under the phase-matching condition. The influences of the external electric field on the polarizations and amplitudes of three interactive waves are studied.

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## 1 Introduction

Periodically poled lithium niobate (PPLN) is a popular nonlinear crystal employing the quasi-phase-matching (QPM) technique, which has several desirable advantages for frequency conversion, such as having a large effective nonlinear coefficient, wide transparent range, and long available crystal length [1, 2]. In PPLN, the sign of the nonlinear coefficient changes periodically owing to the periodic domain structure, and PPLN has been widely used for nonlinear optical interactions, such as second-harmonic generation and optical parametric oscillation [3, 4]. Besides the nonlinear optical coefficient, the electro-optic coefficient is also modulated periodically owing to the periodic domain inversion in PPLN, and the electro-optic effect in PPLN is extensively studied [5–8]. Under a certain condition, the polarization of the light propagating in the crystal can be modulated by the external dc electric field. A Solc-type wavelength filter is proposed and observed [7, 8].

As is well known, the key to QPM is to design a structure that provides a set of reciprocal vectors to compensate for the mismatches of wave vectors owing to the dispersion of refractive index. In general, one-component periodic structure can provide a set of reciprocal vectors to compensate for one-wave vector mismatch. For second-harmonic generation (SHG) the period depends on the dispersion between the fundamental and SH waves; for polarization coupling, the period is determined by the index of the ordinary and ex-

traordinary waves. Because the domain period of PPLN is a function of wavelength as well as temperature, in special cases, periodic structure of PPLN may simultaneously compensate for the phase mismatches of polarization coupling and SHG.

In this paper, wave-coupling equations are proposed including polarization coupling and frequency doubling processes in a single PPLN crystal. Under the QPM conditions for both polarization coupling and SHG, there is a competition between the polarization coupling and SHG with an external dc electric field.

## 2 Theory and simulation

A schematic diagram of the structure is shown in Fig. 1. The propagation direction and the polarization of pump wave are along the  $x$ -axis and  $y$ -axis of the PPLN, respectively, and the dc electric field is applied along the  $y$ -axis.

When the period of the crystal is satisfied for both SHG and polarization coupling in a certain temperature, the  $y$ -polarized pump wave will be coupled to  $z$ -polarized waves by the EO effect, and in the same time, the energy of generated  $z$ -polarized fundamental waves will be transferred to  $z$ -polarized second-harmonic waves by the QPM nonlinear interaction. The propagations of three waves together with the interactions among them are governed by the coupling equations, where EO coupling and the frequency-doubling process are both involved. Although the equations that describe the two processes can be inferred from the literature [9], they cannot be used directly in the present case.

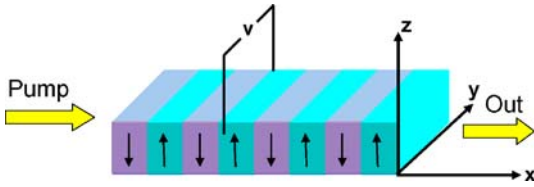
Similarly to [10, 11], stating from Maxwell's equations and employing the plane-wave approximation, the coupling equations ( $\omega_{1y} \rightarrow \omega_{1z}$ ,  $\omega_{1z} + \omega_{1z} \rightarrow \omega_{2z}$ ) including both the EO coupling and frequency doubling can be deduced as follows:

$$\frac{dE_{1y}}{dx} = -i \frac{\omega_1}{n_{1y}c} [\varepsilon_{23}(x)E_{1z}e^{i\Delta k_1 x} - \Delta\varepsilon(x)E_{1y}], \quad (1a)$$

$$\frac{dE_{1z}}{dx} = -i \frac{\omega_1}{n_{1z}c} [\varepsilon_{23}(x)E_{1y}e^{-i\Delta k_1 x} + d_{33}(x)E_{1z}^*E_{2z}e^{i\Delta k_2 x}], \quad (1b)$$

$$\frac{dE_{2z}}{dx} = -i \frac{\omega_2}{2n_{2z}c} d_{33}(x)E_{1z}^2e^{-i\Delta k_2 x}, \quad (1c)$$

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**FIGURE 1** Schematic diagram of the polarization coupling and SHG in the PPLN with an applied electric field

with  $\varepsilon_{23}(x) = -\gamma_{51} E_y n_{1y}^2 n_{1z}^2 f(x)$ ,  $\Delta\varepsilon = -\gamma_{22} E_y n_{1y}^4 f(x)$ ,  $d_{33}(x) = d_{33} f(x)$ ,  $\Delta k_1 = k_{1y} - k_{1z}$ ,  $\Delta k_2 = k_{2z} - 2k_{1z}$ .

The terms  $E_{j\xi}$ ,  $\omega_{j\xi}$ ,  $k_{j\xi}$  and  $n_{j\xi}$  ( $j = 1, 2$ , referring to the fundamental wave and the second harmonic, respectively, and  $\xi = y, z$  represent the polarization) are the electric fields, the angular frequencies, the wave vectors and the refractive indices, respectively. The term  $c$  is the speed of light in vacuum,  $d_{33}$  is the nonlinear coefficient,  $\gamma_{21}$  and  $\gamma_{51}$  are the EO coefficients, and  $f(x)$  is the structure function that is  $+1$  or  $-1$  for the positive or negative domains of the PPLN.

If  $f(x)$  is a periodic function of  $x$  with period  $\Lambda$  due to the periodically modulated nonlinear coefficient or electro-optic coefficient, it can be written as a Fourier series:

$$f(x) = \sum_m g_m \exp(-iG_m x), \quad (2)$$

where  $G_m$  are the reciprocal vectors and  $g_m$  are the amplitudes of the reciprocal vectors. When the phase mismatch in polarization coupling and SHG can be compensated by the reciprocal vectors  $G_1$  and  $G_2$ , the coupling equations (1) can be simplified as:

$$\frac{dA_{1y}}{dx} = iK_1 A_{1z} - iK_3 A_{1y}, \quad (3a)$$

$$\frac{dA_{1z}}{dx} = iK_1^* A_{1y} - iK_2 A_{1z}^* A_{2z}, \quad (3b)$$

$$\frac{dA_{2z}}{dx} = -\frac{i}{2} K_2 A_{1z}^2, \quad (3c)$$

with

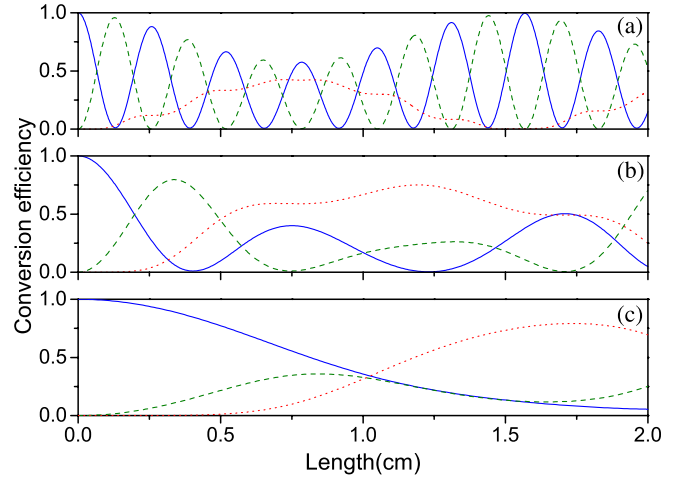
$$A_j = \sqrt{\frac{n_j}{\omega_j}} E_j, \quad K_1 = \frac{\gamma_{51} g_1 \omega_1 E}{c} \sqrt{n_{1y}^3 n_{1z}^3},$$

$$K_2 = \frac{d_{33} g_2}{c} \sqrt{\frac{\omega_1^2 \omega_2}{n_{1z}^2 n_{2z}}}, \quad K_3 = \frac{\gamma_{22} g_1 \omega_1 E}{c} n_{1y}^3,$$

and in this case, the phase mismatches are  $\Delta k_1 = k_{1y} - k_{1z} - G_1 = 0$  and  $\Delta k_2 = k_{2z} - 2k_{1z} - G_2 = 0$ .

Equations (3) can be used to describe the polarization coupling and frequency doubling effects in QPM material with an external electric field applied along the  $y$ -direction.

The numerical calculations with perfect phase-matched conditions are plotted in Fig. 2. The coupling between the polarization coupling and frequency doubling leads to a continuous energy transfer among the  $y$ -polarized pump wave,  $z$ -polarized FH and  $z$ -polarized SH. It is obviously that the conversion efficiencies depend strongly on the ratio of coupling coefficients  $K_1$  and  $K_2$ . Different ratios correspond to different conditions of the energy transfer process. As seen from (3b), the right-hand side of the equality is composed

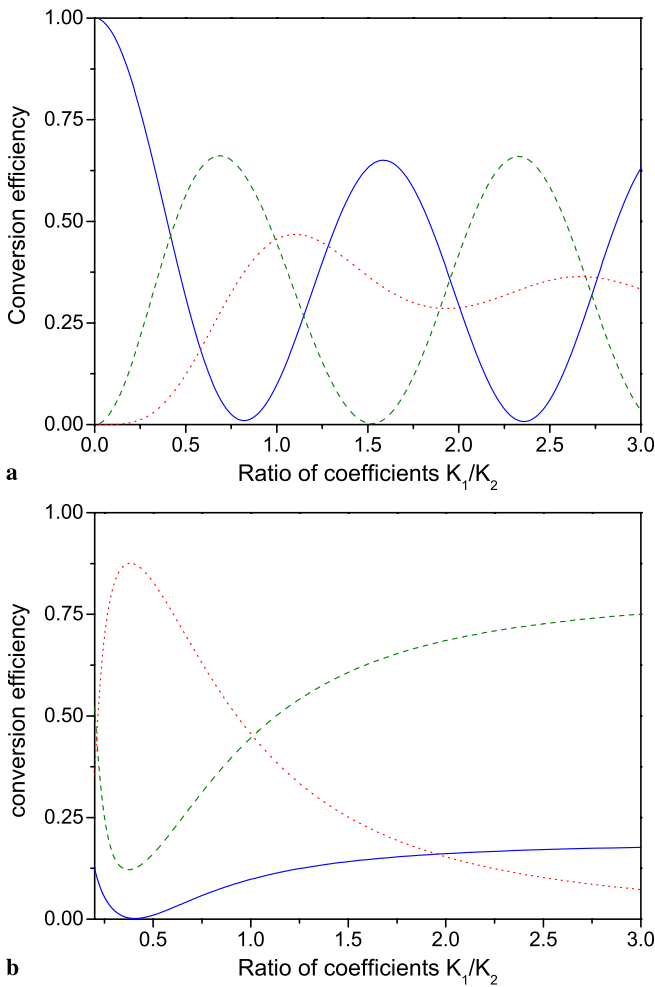


**FIGURE 2** Conversion efficiency vs. the crystal length on the following ratios of two coupling coefficients  $K_1/K_2$ : (a) 3, (b) 1, (c) 0.25. Blue solid, olive dashed and red dotted curves correspond to  $y$ -polarized pump,  $z$ -polarized FH, and  $z$ -polarized SH. Here  $K_2 = 400$ ,  $K_3 = 0.21K_1$  and  $A_{1y}(0) = 1.0$  are assumed

of two terms: one represents the polarization coupling gain and the other describes the loss to generate SH. The competition between the two processes is dependent on the coupling coefficients  $K_1$  and  $K_2$ . Figure 2a shows the situation for  $K_1/K_2 = 3 > 1$ . In this situation, the polarization coupling coefficient  $K_1$  is much larger than the frequency doubling coupling coefficient  $K_2$ , indicating that here the polarization coupling process is more efficient than the frequency doubling process. The energy exchange between the  $y$ -polarized pump wave and  $z$ -polarized FH takes place time after time, at the same time the energy of  $z$ -polarized FH is converted into the SH wave. Moreover, the intensity of SHG improves rapidly when the intensity of  $z$ -polarized FH is relatively high. It can be seen that when the conversion of the SH reaches its maximum, the energy summation of the other waves reaches its lowest spot. After that, the conversion of the SH begin to attenuate. With the decrease of  $K_1$ , the energy exchange between  $y$ -polarized pump wave and  $z$ -polarized FH will go slowly. When  $K_1/K_2 = 1$  (Fig. 2b), numerical simulation shows that the conversion efficient of SH is high after the second coupling process between the  $y$ -polarized pump wave and  $z$ -polarized FH. The intensity of the  $y$ -polarized pump wave and  $z$ -polarized FH is special low after three coupling process, and the pump energy have been transferred to FH and SH completely. When  $K_1/K_2 < 1$  (Fig. 2c), the situation is quite different. Owing to the small ratio of coupling coefficients  $K_1$  to  $K_2$ , the SHG process obviously builds up, while the EO coupling is weakened. In this situation, the conversion efficiency of SH is relatively high due to the QPM condition. The small-signal approximation cannot be used to analyze the SHG process, and the pump depletion should be considered. The  $z$ -polarized SH can be approximated as

$$|A_{2z}(x)| = \frac{|\bar{A}_{1z}|}{\sqrt{2}} \tanh(\sqrt{2}K_2 |\bar{A}_{1z}| x).$$

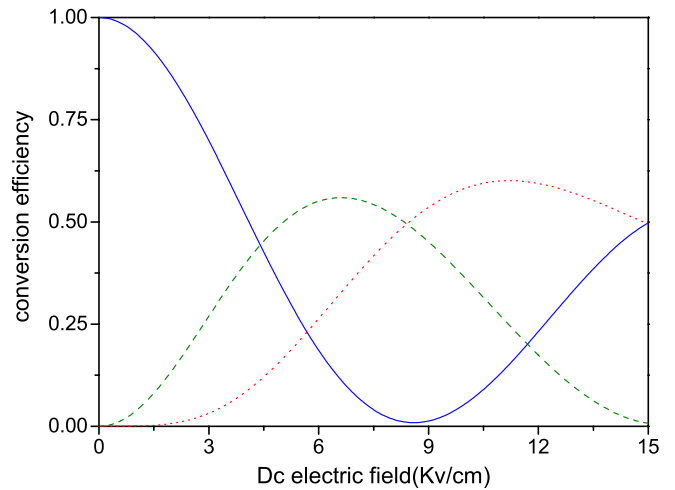
The three-wave output intensity and polarization depend not only on the ratio but also on the value of the normal-



**FIGURE 3** Calculated conversion efficiency vs. the ratio of two coupling coefficients,  $K_1$  and  $K_2$ , with a crystal length of 1 cm. (a)  $K_2 = 400$ ; (b)  $K_1 = 400$ ; here  $K_3 = 0.21 K_1$ , blue solid, olive dashed and red dotted curves correspond to y-polarized pump, z-polarized FH, and z-polarized SH

ized length. In Fig. 3 the three-waves conversion efficiency are plotted as the function of the ratio of the coupling coefficients  $K_1/K_2$  with a length of  $x = 1$  cm. As we know, the frequency doubling coefficient  $K_2$  is fixed when the crystal material and the period are determined, so that the ratio of the coupling coefficients  $K_1/K_2$  can only be shifted by the polarization coupling coefficient  $K_1$ , which is related to the external dc electric field. However, if the external dc electric field cannot be modulated in some special case, we can shift the ratio of the coupling coefficient by properly choosing the crystal material and temperature. Due to the strong dependence of coupling behavior on the coupling coefficient ratio, a simple and convenient method for fast tuning the energy flowing in PPLN can be employed by modulating the external dc electric field.

In one example a special case is assumed, for  $\text{LiNbO}_3$  crystal, when the pump wavelength is set to be 1506.8 nm, the two-wave vector mismatches are  $\Delta k_1 = \Delta k_2 = 0.3069 \mu\text{m}^{-1}$  at the temperature of 26 °C. In this case, a PPLN with the period 20.47  $\mu\text{m}$  and a symmetric duty cycle can be employed, where the reciprocal vector is used to perform QPM. Figure 4 shows the calculated three-wave conversion effi-



**FIGURE 4** Dependence of conversion efficiency on the external dc electric field. The length of the PPLN is 5 mm, and the y-polarized pump intensity is  $100 \text{ MW/cm}^2$ . Blue solid, olive dashed and red dotted curves correspond to y-polarized pump, z-polarized FH, and z-polarized SH, respectively

ciency as a function of the external dc electric field. In the calculation, the length of the PPLN is 5 mm, the pump intensity of y-polarized pump wave is set to be  $100 \text{ MW/cm}^2$ , and the refractive index of lithium niobate for different polarized waves are calculated from the Sellmeier data of [12, 13]. As shown in Fig. 4, when the external dc electric field is increased, the EO coupling process becomes rapid and z-polarized SH is increased along with a reduction of y-polarized pump wave. It can be seen that, when the conversion of the z-polarized fundamental wave reaches its maximum, the y-polarized pump wave is not depleted completely, with the intensity of SH still increasing. When the dc electric field is about 8.4 kV/cm, the z-polarized FH and SH waves have a relatively high energy. When the dc electric field is increased further, the intensity of SH reaches its maximum at the spot of being 11.2 kV/cm. After that, both z-polarized fundamental wave and SH become less efficient, and the energy is transferred back to the y-polarized pump again. From Fig. 4, it can be concluded that the power and the polarization of three waves can be modulated by the applied electric field due to the competition between the SHG and polarization coupling. It should be noted that the working wavelength of the device can be selected by the temperature of the crystal.

### 3 Conclusion

In conclusion, we have generalized the united wave-coupling theory including polarization coupling and SHG in a single PPLN, by which we have studied the influences of polarization coupling coefficient and frequency doubling coefficient on the amplitudes of different light-polarization components. We find that the change of the coupling coefficient ratio will result in different coupling behavior by varying the external dc electric field. In this way, polarization control and frequency conversion can be achieved simultaneously. This technique is particularly interesting for applications of both information transmission and frequency conversion.

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