Improved method for recovering graded-index profile of isotropic waveguide by cubic spline function

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Abstract. An improved method for recovering the refractive index profile of an isotropic graded-index waveguide is presented on the basis of the cubic spline interpolation method. The surface refractive index of the waveguide can be obtained by using a polynomial function that is related with the parameters of all modes. The simulation results of several typical index distributions show that this method can be employed to recover a graded-index profile well. [DOI: 10.1117/1.3463016]

Subject terms: refractive index profile; cubic spline function interpolation; analytical transverse matrix; Gregory–Newton interpolation.

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1 Introduction

An optical waveguide plays an essential role in optical integrated devices. The refractive index profile, which is one of main characteristics of the optical waveguides, offers significant propagation information.

In past years, several methods, especially nondestructive ones, have been created and developed to recover the gradient refractive index profile on the condition that the effective refractive indices of each guided mode have already been measured, such as reflectivity profiling,[1] ellipsometry,[2] Wigner distribution of an optical waveguide.[3–5] Among them, the most popular methods are the inverse WKB method,[6–8] and the improved IWKB method.[9] The inverse analytical transfer matrix (ATM) was represented afterward by Ding et al.[10] who induced the ATM method[11] into the iterative method.

Recently, a new method for recovering the refractive index profile of an anisotropic graded-index waveguide from the effective indices by using a cubic spline interpolation function was proposed by Liao et al.[12] The method not only ensures a smooth index profile, which is consistent with practical graded-index distributions, but also uses an exact dispersion equation to verify the interpolated in iterative method.

In this paper, some improvements are made to make this method more theoretically reliable and practically applicable. There is no tentative parameter, and the first differential order of refractive the index profile at the surface is needed in recovering the index profile, which makes the recover result closer to the real refractive index distribution of optical waveguide. Compared to the results in Ref. 5, a more accurate surface refractive index is obtained.

2 Mathematical Improvement

In the general cubic spline method, the boundary condition is the first-order differential, given compulsively and arbitrarily, which unavoidably brings some uncertainty into the recovering process. We change it into the nature boundary condition in the process of cubic spline interpolation, which means the values of the second-order differential at the two boundaries compulsively equals zero. Because the second-order differential will bring much less error than the first-order differential, the cubic spline method promises accuracy during the mode points. Thus, we get the function of refractive index of depth x as

$$n_i(x) = \frac{1}{6}\frac{(x_{i+1}-x)^3}{x_{i+1}-x_i}n''_i + \frac{1}{6}\frac{(x_i-x)^3}{x_{i+1}-x_i}n''_{i+1} + \left[n_{i+1} - \frac{1}{6}n''_{i+1}(x_{i+1}-x_i)^2\right]\frac{x-x_i}{x_{i+1}-x_i} + \left[n_i - \frac{1}{6}n''_i(x_{i+1}-x_i)^2\right]\frac{1}{x_{i+1}-x_i},$$

$$x_i \leqslant x \leqslant x_{i+1},$$

$$i = 0 \cdots k. \tag{1}$$

In order to obtain the unknown parameter of the refractive index profile function above, the equation is given by letting the values of the first-order differential and second-order differential continuous at the interval as

$$\frac{x_i - x_{i-1}}{6}n''_{i-1} + \frac{x_{i+1}-x_{i-1}}{3}n''_i + \frac{x_{i+1}-x_i}{6}n''_{i+1} = \frac{n_{i+1}-n_i}{x_{i+1}-x_i} - \frac{n_i - n_{i-1}}{x_i - x_{i-1}}, \quad i = 1 \ldots k. \tag{2}$$

In addition, two additional natural boundary conditions, $y''_0=0$ and $y''_{k+1}=0$, are introduced into the solving process, which is more universal compared to the second boundary condition in Ref. 5.

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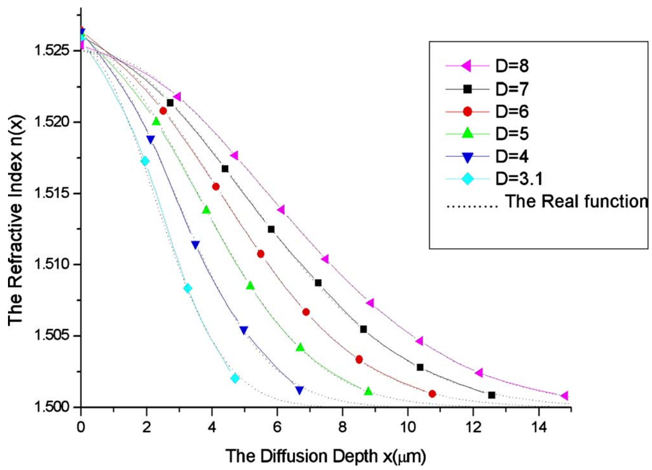

Fig. 1 The gauss refractive index profile recovered by improved cubic spline function interpolation method at the wavelength at 632.8 nm. D varied from 3.1 to 8 μm, allowing 3–8 modes.

The group of equations can be converted into a triangle matrix equation. Thus, it is easy to get all the values of the second-order differential at the mode points. By substituting them into Eq. (1), a smooth refractive index profile of the tested sample can be obtained.

The value of the surface refractive index in the former method is calculated by the cubic function derived from the zero mode and first mode at the depth of zero in Ref. 5. The first two modes do not include the information of all the modes tested by experiment. The change that we made is to connect the surface refractive index to all modes by using the polynomial function. Suppose the tested sample has a number of $k+1$ modes, then there exists a function as follows:

$$n(x) = \sum_{i=0}^{k+1} a_i x^i, \tag{3}$$

and each mode is the point of this function; thus, the $k+1$ modes provide us with $k+1$ equations as

$$\sum_{i=0}^{k+1} a_i x_j^i = n_j \quad j = 0 \ldots k. \tag{4}$$

The one additional equation we supposed to solve all the a_i is that

$$\left. \frac{d^2}{dx^2} \sum_{i=0}^{k+1} a_i x_j^i \right|_{x=0} = 0 \quad j = 0 \ldots k. \tag{5}$$

Equation (5) originated from the nature boundary condition motioned earlier, which ensures that the value of the second-order differential at the surface point equals zero just as the cubic spline function's extra nature boundary condition. With these $k+2$ equations, all a_i are determined. Therefore, the value of the surface refractive index can be easily obtained by letting $x=0$.

In this method, we determine the parameters of the decay term from the cubic function calculated before the last mode. This improvement will change greatly during a series of iterative loop. Supposed the decay term is in the form of $n(x)=Ns+b_1 e^{-b_2 x}$, where b_1 and b_2 are two parameters that need to be determined. From the calculation above, the value of the first-order differential at the last mode is known to be as follows:

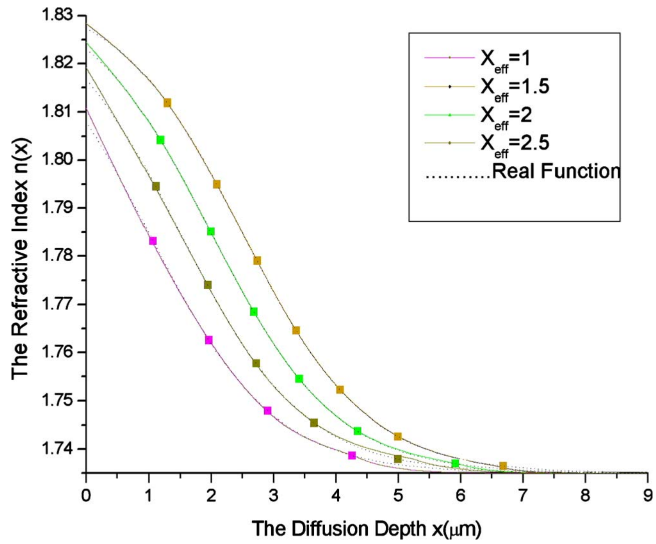

Fig. 2 The slow Fermi refractive index profile recovered by improved cubic spline function interpolation method at the wavelength at 632.8 nm. And x_{eff} varied from 1 to 2.5 μm, allowing 4–7 modes.

$$\begin{aligned} n_i'(x) = &-\frac{1}{2}\frac{(x_{i+1}-x)^2}{x_{i+1}-x_i} n_i'' + \frac{1}{2}\frac{(x_i-x)^2}{x_{i+1}-x_i} n_{i+1}'' \\ &+ \left[n_{i+1} - \frac{1}{6} n_{i+1}''(x_{i+1}-x_i)^2 \right] \frac{1}{x_{i+1}-x_i} \\ &- \left[n_i - \frac{1}{6} n_i''(x_{i+1}-x_i)^2 \right] \frac{1}{x_{i+1}-x_i}. \end{aligned} \tag{6}$$

By letting both $n(x)$ and $n'(x)$ be continuous at the last mode point, which is also the turning point from the propagating part to the decay part, we can obtain the formula for these two parameters as follows:

$$b_1 = \frac{n'(x_k)}{Ns - n(x_k)} \quad b_2 = \log[-n'(x_k)]. \tag{7}$$

During the ATM process, the total index profile is departed into several parts, but the value of each part is obtained from its left boundary value. Instead, in the ATM process, we use the average of the left and right boundary values, thus avoiding successive appearance of the situation $\kappa=0$ at the denominator, which often holds back the proper running of the ATM process in Ref. 5. Besides, during the circle of the iterative process, a relaxation parameter w is introduced to ensure that the iterating process keeps running, not only toward a convergent direction, but also with a fast speed.

3 Recovery of Typical Functions

For verifying the reliability of the proposed method for recovering the index profile, some typical distribution func-

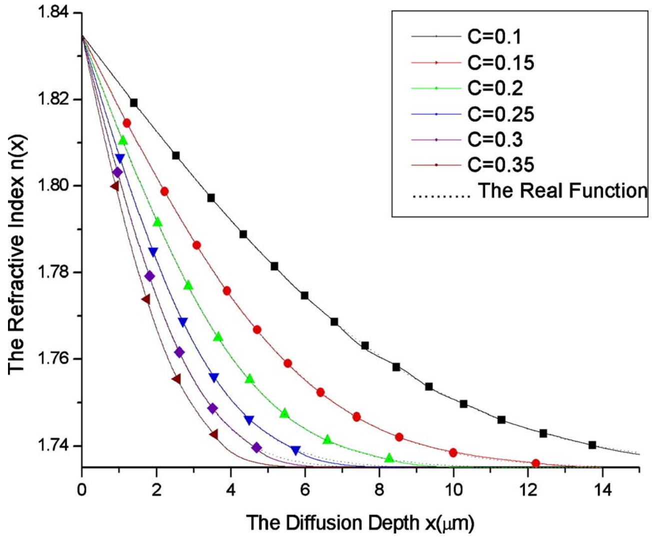

Fig. 3 The error function refractive index profile recovered by improved cubic spline function interpolation method at the wavelength at 632.8 nm. *C* varied from 0.1 to 0.35, allowing 4–14 modes.

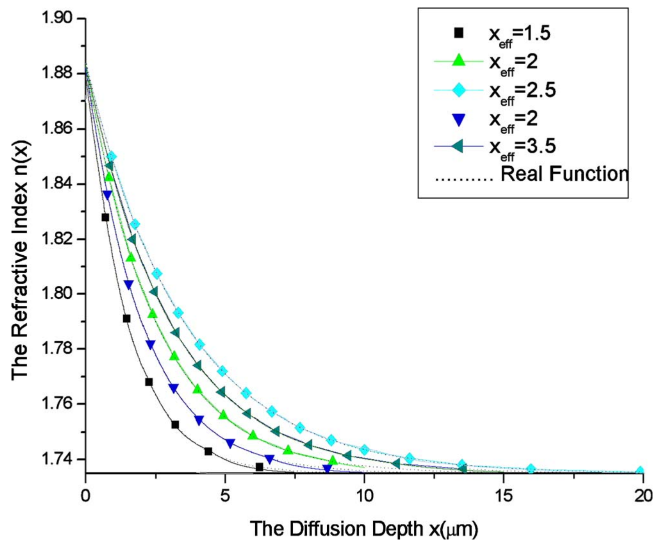

Fig. 4 Exponential function refractive index profile recovered by improved cubic spline function interpolation method at the wavelength at 632.8 nm. x_{eff} varied from 1.5 to 3.5 μm, allowing 6–15 modes.

tion representing some index profile is given to test this method at the wavelength of 632.8 nm, such as Gaussian, error function, Fermi, and exponential profile.

First, we supposed the refractive index profile is in the distribution of the Gaussian profile, which is described as

$$n(x) = 1.5 + 0.025 \exp(-x^2/D^2). \tag{8}$$

The parameter D varied from 3.1 to 8 μm, and waveguides with such a profile could support three to eight modes. According to this given index distribution, we can calculate the exact effective index. The results are shown in Fig. 1. We can see that the recover index profile is almost the same as the exact function profile. We should point out that there is a very small error at the surface point.

Second, we supposed a slow Fermi-distributional refractive index profile as

$$n(x) = 1.735 + \frac{0.1}{1 + e^{x - x_{\mathrm{eff}}}}. \tag{9}$$

From the calculated effective index, we reconstruct its profile by the proposed method, As shown in Fig. 2. The recover profiles are very close to the original function.

The error function is in the form of

$$n(x) = 1.735 + 0.1\,\mathrm{erfc}(C \cdot x). \tag{10}$$

The recover results for error function are shown in Fig. 3. It is clear that the profile recovered by our method is almost overlapped to the original function.

At last, the situation of the exponential index distribution is displayed in Fig. 4. We suppose that exponential function of index profile is

$$n(x) = 1.735 + 0.15 e^{-(x/x_{\mathrm{eff}})}. \tag{11}$$

The recovered profile is very close to the original one, with the parameter of x_{eff} various from 1.5 to 3.5.

From Figs. 1–4, it was determined that the recover results fit the supposed refractive index distributions perfectly. Compared to the results in Ref. 5, no tentative parameter and the first differential order of refractive index profile at the surface is needed in recovering index profile. Although the index profile we used for verifying our proposed method is a mathematic function instead of a practical one, we believe that it can be employed to the actual graded-index profile with several modes. As we know, waveguides fabricated by ion exchanging, ion implantation, annealing, or fabrication during material growth have graded-index profiles which should be someone of the typical functions we used in the calculation. It is reasonable that the real index profile can be decomposed into linear superosition of the several typical functions. Thus, we can reconstruct the real profile as well as recovering the index profile with typical functions.

Table 1 The effective refractive index for TE mode measured by M-line method.

λ (μm)	632.8	632.8	632.8
Mode	Sample 1	Sample 2	Sample 3
0	1.523728627	1.521643522	1.527878159
1	1.516576333	1.517978141	1.523728627
2	—	1.51499571	1.519726162
3	—	—	1.515874302
Substrate index	1.51432	1.51432	1.51432

Table 2 The effective refractive index for TE mode calculated by current method and method in Ref. 5.

	Sample1		Sample 2		Sample 3	
Mode	Current method	Method in Ref. 5	Current method	Method in Ref. 5	Current method	Method in Ref. 5
0	1.523728627	1.52373249	1.521643009	1.52164352	1.527878159	1.52787003
1	1.516576333	1.51657633	1.517978141	1.51797814	1.523728627	1.52372863
2	—	—	1.514995710	1.51500831	1.519726162	1.51972616
3	—	—	—	—	1.515874302	1.51588991
Surface index	1.529645302	1.53087863	1.525312383	1.52473352	1.531875206	1.53119816

4 Experimental Data on Profile Recovery

In contrast to the method in Ref. 5, copper ion-exchange planar waveguides were made by the ion-exchange method. In Table 1, the effective refractive index for the transverse electric (TE) mode of the samples were measured by M-line method.[13] In Table 2, the effective refractive index for TE mode were calculated by the current method and the method in Ref. 5 respectively. In Fig. 5, the refractive index distributions were recovered by the current method and the method in Ref. 5. Under the same step conditions (Step =0.002), the calculation accuracy was improved by an order of magnitude, which was 10^{-6} for current method and 10^{-5} for the method in Ref. 5 from Tables 1 and Table 2.

In Fig. 5, it was determined that the recover result of the current method was similar with the one from Ref. 5, when the refractive index distribution was gentle, such as simple 2 and simple 3. But when the refractive index profile was steep, the recover result of current method was different with the one of Ref. 5, such as simple 1 in Fig. 5. The difference was originated from the few number of effective refractive index and the steep refractive index distribution.

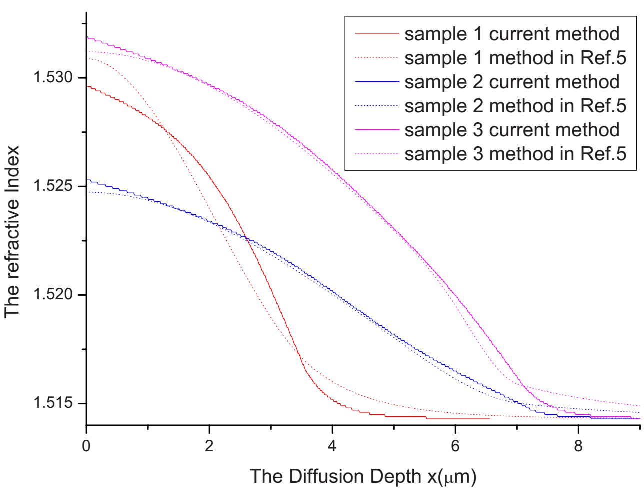

Fig. 5 Refractive index distributions of samples recovered by improved cubic spline function interpolation method and the method in Ref. 5 at the wavelength at 632.8 nm.

5 Conclusion

The improved cubic spline method is demonstrated to recover the monotomic refractive index of the graded-index waveguide smoothly with good accuracy. With the nature boundary condition of the cubic spline, an extra uncertain parameter is omitted in the calculation process. The surface refractive index of the waveguide can be obtained by using a polynomial function that is related with the parameters of all modes. The method is applied in the waveguide whose index profiles are described by some typical functions, and the results show that this method comes closer to the actual situations.

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References

1. J. Steffen, A. Neyer, E. Voges, and N. Hecking, "Refractive index profile measurement techniques by reflectivity profiling: vidicon imaging, beam scanning, and sample scanning," *Appl. Opt.* **29**(30), 4468–4472 (1990).
2. D. Tonova, A. Paneva, and B. Pantchev, "Determination of refractive index profiles of gradient optical waveguides by ellipsometry," *Opt. Commun.* **150**(1–6), 121–125 (1998).
3. D. Dragoman, M. Dragoman, and J. Meunier, "Recovery of the refractive-index profile from the Wigner distribution of an optical waveguide," *Appl. Opt.* **37**, 2357–2360 (1998).
4. B. Chen, H. Hamanaka, and K. Iwamura, "Recovery of refractive-index profiles of planar graded-index waveguides from measured mode indices: an iteration method," *J. Opt. Soc. Am. A* **9**(8), 1301–1305 (1992).
5. D. Sarid, "Recovery of the refractive-index profile of an optical waveguide from the measured coupling angles," *Appl. Opt.* **19**(10), 1606–1608 (1980).
6. J. White and P. Heidrich, "Optical waveguide refractive index profiles determined from measurement of mode indices: a simple analysis," *Appl. Opt.* **15**(1), 151–155 (1976).
7. P. Hertel and H. Menzler, "Improved inverse WKB procedure to reconstruct refractive index profiles of dielectric planar waveguides," *Appl. Phys. B* **44**(2), 75–80 (1987).
8. P. Mathey and P. Jullien, "Numerical analysis of a WKB inverse

method in view of index profile reconstruction in diffused waveguides," *Opt. Commun.* **122**(4–6), 127–134 (1996).
9. K. Chiang, "Construction of refractive-index profiles of planar dielectric waveguides from the distribution of effective indexes," *J. Lightwave Technol.* **3**(2), 385–391 (1985).
10. Y. Ding, Z. Cao, and Q. Shen, "Determination of optical waveguide refractive-index profiles with the inverse analytic transfer matrix method," *Opt. Quantum Electron.* **36**(6), 489–497 (2004).
11. Z. Cao, Y. Jiang, Q. Shen, X. Dou, and Y. Chen, "Exact analytical method for planar optical waveguides with arbitrary index profile," *J. Opt. Soc. Am. A* **16**(9), 2209–2212 (1999).
12. W. Liao, X. Chen, Y. Chen, and Y. Xia, "Index profiling of anisotropic graded-index planar waveguides from effective indices," *J. Opt. Soc. Am. A* **22**(7), 1334–1340 (2005).
13. P. Tien and R. Ulrich, "Theory of prism-film coupler and thin-film light guides," *J. Opt. Soc. Am.* **60**(10), 1325–1337 (1970).

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