Control of spin angular momentum in quasi-phase-matched material

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A method to control spin angular momentum (SAM), based on mutual effects of a electro-optic effect and a quasi-phase-matched (QPM) technique in QPM material, is proposed. By controlling the external electric field or operating wavelength, the transfer between left- and right-handed circularly polarized photons is achieved, thus the total SAM is manipulated. The external electric field needed in this method for the complete modulation of the total SAM per photon from 0 to $\hbar$ is as low as $0.44\,\mathrm{kV/cm}$. This method will see its applications in micromanipulation by light. © 2011 Optical Society of America

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1. Introduction

As with other physical realities, the light also carries energy, linear momentum (LM), and angular momentum (AM). Generally, the AM of the light field could be separated into the orbital angular momentum (OAM) and spin angular momentum (SAM) [1]. OAM is related to the helical phase fronts of light beams [2]. While the SAM is associated with circular polarization and arises from the spin of an individual photon [3]. Each photon carries a SAM of $\hbar$ or $-\hbar$ for the left-handed circular polarized (LHCP) light or right-handed circular polarized (RHCP) light, respectively. When the RHCP light is transformed into LHCP light by a birefringent wave plate, each photon has a transformation of SAM by $2\hbar$, and the measurement of the consequent torque exerted on the wave plate *Beth* has proved the existence of SAM [4]. Interaction between light and matter is accompanied with the exchange of LM, leading to a net force, and AM, which induces a rotational force [5,6]. Usually, this delicate optical force is all but imperceptible for a macroscopic object but can have a substantial influence on the microscopic or mesoscopic one. The micromanipulation, resulting from absorption or transformation of SAM from the photons in the beam, has been intensively investigated for transparent birefringent particles, metallic particles, and molecular particles [5,7–9]. Apparently, a convenient control of SAM will play a significant role in the area of manipulation in microscopic or mesoscopic scales. The media with anisotropy, as in the birefringent wave plate and trapped microparticles, have been extensively used to control the SAM of light [4,5]. However, the modulation progresses of those methods are not continuous, which is inconvenient for macro-operation. Recently,Chen *et al.* presented a convenient method to manipulate SAM in an optically active crystal based on Pockel and Faraday effects by adjusting the external electric and magnetic field [10,11]. However, due to the small electro-optic coefficient, the electric field required in their method is so high that they may damage the media. Meanwhile, the quasi-phase-matched (QPM) material, a kind of periodically poled ferroelectric crystal (e.g. periodically poled lithium niobate (PPLN)), has been extensively used for polarization manipulation

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[12–15], in which the sign of the electro-optics coefficients are periodically modulated due to periodical domain inversion [14], and the ordinary and extraordinary light passing through it are analyzed by QPM theory [16], which we called the QPM technique. In this article, we propose an approach based on the mutual effect of the QPM technique in QPM material and its transverse electro-optic effect to manipulate the SAM continuously by consecutive adjustment of the external electric field and operating wavelength. The electro-optic effect of crystal has been extensively used to manipulate the polarization states of light. As shown in Fig. 1(a), when the external electric field is applied along the Z axis of the monodomain crystal with the symmetry of 3 m, called the longitudinal electro-optic effect, the amplitude of the index ellipsoid changes with the external electric field [17]. While the external electric field is applied along the Y axis of the monodomain crystal shown in Fig. 1(b), called the transverse electro-optic effect, the index ellipsoid rotates a small angle about the X axis, which is proportional to the external electric field [17]. When the external electric field is applied along the Y axis of PPLN, shown in Fig. 1(c), the index ellipsoid rotates in the reverse direction in a positive domain and negative domain because of different signs of the electro-optic tensor. Because of this periodic inverse rotation of the index ellipsoid along the crystal, the coupling between ordinary and extraordinary light is significantly enhanced. Meanwhile, the ordinary and extraordinary light experience differs among the velocities, thus the polarization state of the light at the output facet of PPLN is very sensitive to the external electric field. The QPM technique enables the operating wavelength to be an effective means to manipulate the state of light. When operating at a QPM wavelength, which satisfies the QPM condition, the polarized direction of light will rotate with an increase of the external electric field, but remain linear polarized thus the SAM is unchanged, which means the electro-optic effect does not influence the SAM in this condition [13]. However, when operating at the wavelength that does not satisfy the QPM condition, which we call the NQPM wavelength, the SAM experiences evolution with the external electric field remarkably.

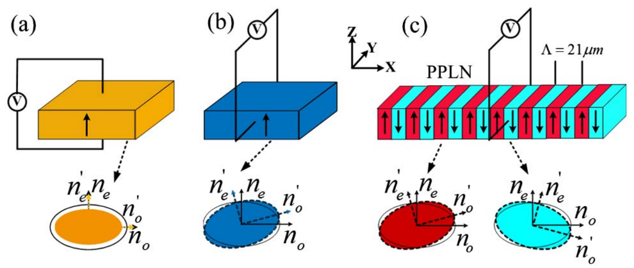

Fig. 1. (Color online) Schematic diagram of the electro-optic effect in crystal. (a) The longitudinal electro-optic effect in the monodomain crystal. (b) The transverse electro-optic effect in the monodomain crystal. (c) The transverse electro-optic effect in PPLN with a period of 21 μm.

In this condition, the kind of region, which has the same sign of SAM and we called the "spin cell," is formed, and it determines the rotational direction of particles when the light is used as a spanner [6]. In addition, the SAM behaves disparately for different incident conditions.

2. Theoretical Analysis

The light at the output facet of PPLN can be expressed as the superposition of the LHCP and RHCP ones or the superposition of the ordinary and extraordinary ones:

$$E(L) = \left\{ E_{\mathrm{lef}} \begin{bmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{bmatrix} + E_{\mathrm{rig}} \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix} \right\} = \begin{bmatrix} E_1(L) \\ E_2(L) \end{bmatrix}, \tag{1}$$

where E_{lef}, E_{rig}, E_1, and E_2 are the complex amplitudes of the LHCP, RHCP, ordinary, and extraordinary light, respectively; L is the length of PPLN. The coupled wave equations of ordinary and extraordinary waves are given by [14]

$$\begin{cases} dA_1/dz = -i\kappa A_2 \exp(i\Delta\beta z) \\ dA_2/dz = -i\kappa^* A_1 \exp(-i\Delta\beta z) \end{cases}, \tag{2}$$

with $\Delta\beta = (k_2 - k_1) - G_m$, $G_m = 2\pi m/\Lambda$, and $\kappa = -\frac{\omega}{2c} \frac{n_1^2 n_2^2 \gamma_{51} E_y}{\sqrt{n_1 n_2}} \frac{i(1-\cos m\pi)}{m\pi}$ $(m = 1, 3, 5 \ldots)$. Here, A_1 and A_2 are the normalized amplitudes of ordinary and extraordinary light, respectively; k_1 and k_2 are the ordinary and extraordinary wave vectors, respectively; G_m is the mth reciprocal vector corresponding to the periodicity of poling; Λ is the period of PPLN; n_1 and n_2 are the refractive indices of ordinary and extraordinary waves, respectively; γ_{51} is the electro-optic coefficient; and E_y is the external electric field intensity. By solving the coupled-mode equation, substituting $E_1(z) = \sqrt{w/n_1} A_1(z) \exp(ik_1 z)$ and $E_2(z) = \sqrt{w/n_1} A_2(z) \exp(ik_2 z)$, and considering $E_{\mathrm{lef}} = \sqrt{1/2}(E_1 + iE_2)$ and $E_{\mathrm{rig}} = \sqrt{1/2}(E_1 - iE_2)$, we obtain

$$\begin{aligned} E_{\mathrm{lef}} = \{&[\cos(sL) + (-i\Delta\beta \\ &+ 2\kappa^* \exp(-i\Delta\beta L)\sqrt{n_1/n_2}) \sin(sL)/(2s)]E_1(0) \\ &+ [(-2ik\sqrt{n_2/n_1} - \Delta\beta \exp(-i\Delta\beta L)) \sin(sL)/(2s) \\ &+ i\cos(sL) \exp(-i\Delta\beta L)]E_2(0)\} \\ &\times \sqrt{2}/2 \exp[i(\Delta\beta/2)L], \end{aligned} \tag{3}$$

$$\begin{aligned} E_{\mathrm{rig}} = \{&[\cos(sL) + (-i\Delta\beta \\ &- 2\kappa^* \exp(-i\Delta\beta L))\sqrt{n_1/n_2} \sin(sL)/(2s)]E_1(0) \\ &+ [(-2i\kappa\sqrt{n_2/n_1} + \Delta\beta \exp(-i\Delta\beta L)) \sin(sL)/(2s) \\ &- i\cos(sL) \exp(-i\Delta\beta L)]E_2(0)\} \\ &\times \sqrt{2/2} \exp[i(\Delta\beta/2)L], \end{aligned} \tag{4}$$

with $s = \sqrt{\Delta\beta^2 + \kappa\kappa^*}$, where $E_1(0)$ and $E_2(0)$ are the ordinary and extraordinary electric fields of the incident light, respectively.

On the basis of quantum theory, each photon possesses energy of $\hbar\omega$, hence the numbers of the LHCP and RHCP photon transmitted at the output surface per unit area per second are the respective average Poynting energy flow divided by $\hbar w$, i.e.,

$$N_{\rm lef} = c\varepsilon_0 |E_{\rm lef}|^2 / 2\hbar w, N_{\rm rig} = c\varepsilon_0 |E_{\rm rig}|^2 / 2\hbar w. \quad (5)$$

Considering each LHCP photon contains the AM of $\hbar$, and the RHCP one $-\hbar$, the total SAM transmitted at the output surface per unit area per second is [1,4]

$$M = c\varepsilon_0 (|E_{\rm lef}|^2 - |E_{\rm rig}|^2) / 2w. \quad (6)$$

Without losing generality, a 2.1 cm long PPLN with a period of 21 μm is considered to investigate the influences of the operating wavelength and external electric field to $N_{\rm lef}$, $N_{\rm rig}$, and the total SAM. Set $m = 1$, corresponding the QPM wavelength of 1.540 μm, and $\gamma_{51} = 32.6$ pm/V. Substituting all these parameters into Eqs. (5) and (6), we can calculate $N_{\rm lef}$, $N_{\rm rig}$, and M. Three incident states, the linearly polarized light along the Y axis (ordinary incidence), along the Z axis (extraordinary incidence), and rotated 45° about the Y axis, are considered.

The normalized LHCP and RHCP photon numbers and the total SAM of the output light with the incident light linearly polarized along the Y axis are shown in Fig. 2. From Figs. 2(a) and 2(b), we find that the distributions of the LHCP and RHCP photon numbers are both centrosymmetric, and their superposition will lead to a unitary value (T), which means the total number of photon is conserved, due to the assumption of invariance of energy during the propagation. The manipulations of the operating wavelength (M_1) and the external electric field (M_2) are accompanied with the transformation of photons between the left- and right- handed ones. Figure 2(c) shows that the total SAM varies with the operating wavelength and external electric field. At the QPM wavelength (L_1), each domain of PPLN acts as a half-waveplate, and the light would remain linearly polarized, hence the total SAM, equal to the incident value, remains zero [18]. When operating at the NQPM wavelength (L_2), the total SAM alters with the external electric field, the effect of which is related to the degree of departure of the operating wavelength from the QPM wavelength. For example, when operating at 1539.85 nm, the transfer of SAM with $\hbar$ for each photon is viable with a 1.232 kV/cm external electric field applied, and for 1540.26 nm, only 0.44 kV/cm is needed. At the fixed external electric field (L_3), adjustment of the wavelength of incident light is also a convenient means to manipulate the SAM. For instance, at 0.44 kV/cm, a 0.52 nm variance of operating wavelength induces a transfer of $2\hbar$ for each photon. It is very interesting to notice that some regions exist, which possess the same sign

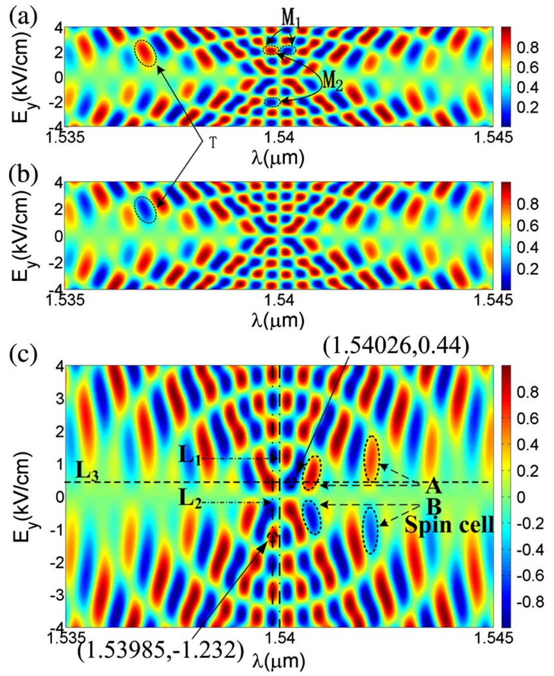

Fig. 2. (Color online) Normalized photon numbers of (a) LHCP light, (b) RHCP light, and (c) the total SAM of the output light, which are controlled by an external electric field and operating wavelength with the incident light linearly polarized along the Y axis. The numbers of the LHCP and RHCP photon are modulated by an operating wavelength (M_1) and external electric field (M_2). The total number of photons is conserved (T). The spin cell (A and B) is formed.

of SAM (A and B), and we call it the spin cell. It has been proven that the rotational direction of particles induced by circularly polarized light, as a spanner, is dependent on the sign of SAM of light [6]. This kind of spin cell would see its splendid application in the micromanipulating field.

The total SAM of the incident light linearly polarized along the Z axis is shown in Fig. 3(a) and rotated 45° about the Y axis as shown in Figs. 3(b)–3(d). By making a comparison of Figs. 2(c) and 3(a), we find that the modulation of extraordinary incidence is similar to ordinary incidence, but experiences inverse progress. Figure 3(b) shows, for linearly polarized incidence rotated 45° about the Y axis, the distribution of SAM is no longer centrosymmetric but axisymmetric about the axis of $Ey = 0$ kV/cm, which means, for arbitrary fixed operating wavelength, the influence of a positive external electric field is equivalent to the reversed one. When operating at a QPM wavelength, the total SAM remains zero. In the near NQPM wavelength condition, such as 1539.83 nm (L_1), 1540.17 nm (L_2), and 1540.88 nm (L_3), we find the transformation of the total SAM carries through periodically with the increment of the external electric field, clearly shown in Fig. 3(c). From Fig. 3(d), we find the SAM changes periodically with the variance of an operating wavelength at the fixed external electric field, 0.792 kV/cm (L_4) and 1.648 kV/cm (L_5).

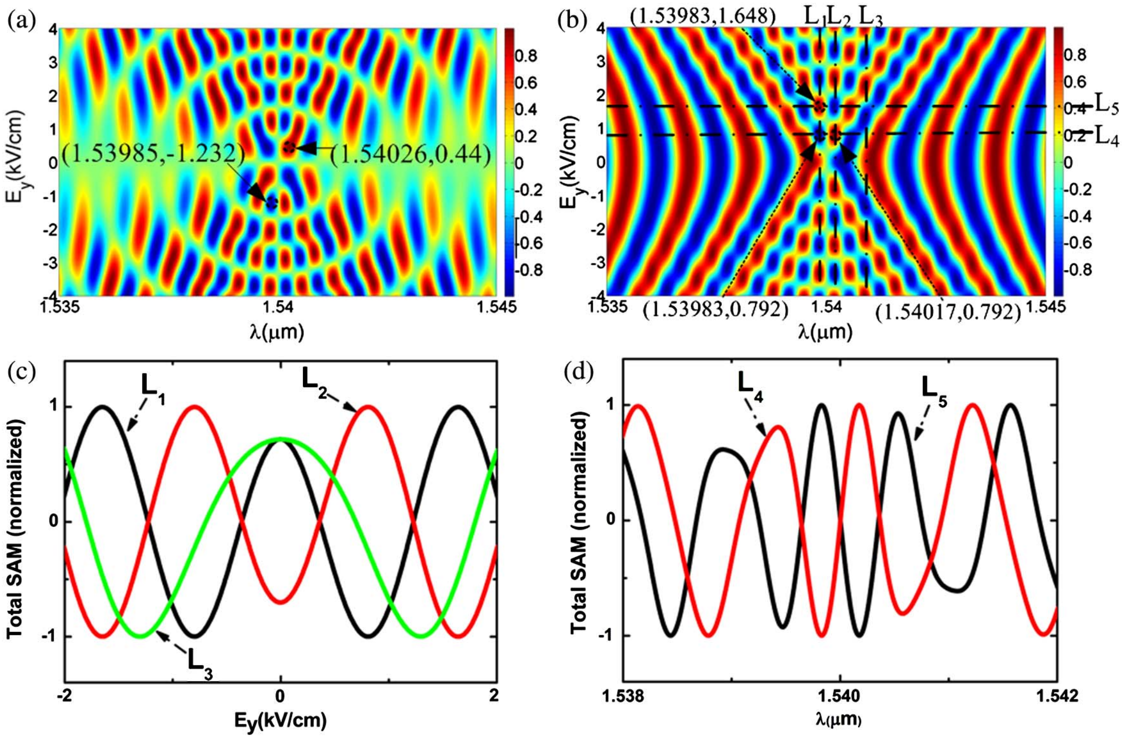

Fig. 3. (Color online) Total SAM (normalized) of the output light controlled by the external electric field and operating wavelength. (a) Linearly polarized incidence along the Z axis. (b) Linearly polarized incidence rotated 45° about the Y axis. (c) and (d) Total SAM of the output light carries through periodically with the electric field at different operating wavelengths and operating wavelengths at the different electric fields in (b), respectively.

3. Experimental Result and Discussion

Our previous experiment has proven that the polarization state evolves on the surface of the Poincare sphere with the change of an external electric field at different operating wavelengths, shown in Fig. 4(a) [13]. The projection of the state on the S_3 axis of Poincare sphere, corresponding to the normalized power of LHCP light, experiences change with external manipulation, hence the total SAM is modulated, according to Eqs. (5) and (6). That the external electric field needed in the experiment is a little larger than the theoretical one arises from the inner electric field existing in such a periodically poled ferroelectric domain structure that may be caused by the strain-optic effect produced in the process of polarization or the photovoltaic effect engendered by the input light [19,20]. In general, the method for manipulation of the SAM proposed here is viable.

It is worth emphasizing that the transition of SAM is consecutive, which means the continuous manipulation of SAM and dynamic control of particles are available by changing the external electric field and operating wavelength. The phase modulators on the market, mainly based on the longitudinal electro-optic effect of lithium niobate crystal, can also be used as SAM controllers. However, the external electric field applied on the lithium niobate monodomain crystal with a length of 2.1 cm for the modulation of SAM of light from $-\hbar$ to $\hbar$ is about 7.23 kV/cm, which

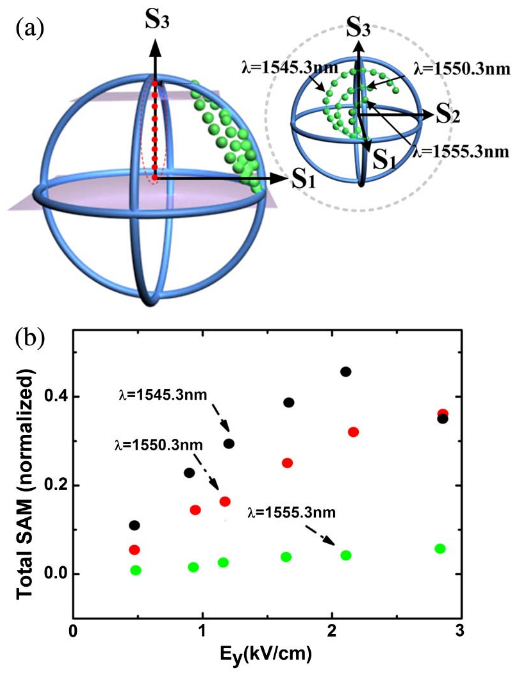

Fig. 4. (Color online) Experimental results of the polarization state evolution induced by the change of the external electric field for a different operating wavelength with incident light linearly polarized along the Z axis [13]. The corresponding transformation of projection on the S_3 axis of Poincare sphere with the increment of external electric field reveals the modulation of the SAM. (a) Schematic diagram of projection on th eS_3 axis of Poincare sphere. (b) The total SAM changes with the external electric field.

is much higher than the 0.44 kV/cm in the PPLN crystal with the same length. The electric field needed in our method is also much lower than that in the Chen *et al.* article, about 66 kV/cm with crystal as long as 1 cm. Because the PPLN consists of thousands of cascaded waveplates with micrometer thickness, it has advantages of being compact and requiring ultralow voltage, compared with the conventional multiple bulk crystalline waveplates, i.e., lithium niobate crystal waveplates. Meanwhile, the SAM of each photon, unlike energy, is independent of frequency, for the LHCP photon $\hbar$ and PHCP photon $-\hbar$, so the modulation of the wavelength is a convenient means for SAM management [4]. The investigations on manipulation in the microscale and spin-orbital AM entanglement based on this approach are necessary in the future [21].

4. Conclusion

In summary, we have theoretically and experimentally shown that the external electric field and operating wavelength could be used to modulate the SAM completely in the QPM material continuously, based on the mutual effect of the QPM technique and electro-optic effect. Because of the advantages of this method, it will see significant application in mesoscopic or microscopic manipulating systems and bioscience. Meanwhile, this result also triggers a wide range of interest and opens a perspective toward the QPM technique.

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References

1. S. Barnett, "Optical angular-momentum flux," J. Opt. B Quant. Semiclass. Opt. **4**, S7–S16 (2002).
2. L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," Phys. Rev. A **45**, 8185–8189 (1992).
3. J. Poynting, "The wave motion of a revolving shaft, and a suggestion as to the angular momentum in a beam of circularly polarised light," Proc. R. Soc. A **82**, 560–567 (1909).
4. R. A. Beth, "Mechanical detection and measurement of the angular momentum of light," Phys. Rev. **50**, 115–125 (1936).
5. M. E. J. Friese, T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein-Dunlop, "Optical alignment and spinning of laser-trapped microscopic particles," Nature **395**, 348–350 (1998).
6. A. T. O'Neil, I. MacVicar, L. Allen, and M. J. Padgett, "Intrinsic and extrinsic nature of the orbital angular momentum of a light beam," Phys. Rev. Lett. **88**, 53601 (2002).
7. A. T. O'Neil and M. Padgett, "Three-dimensional optical confinement of micron-sized metal particles and the decoupling of the spin and orbital angular momentum within an optical spanner," Opt. Commun. **185**, 139–143 (2000).
8. B. Piccirillo, C. Toscano, F. Vetrano, and E. Santamato, "Orbital and spin photon angular momentum transfer in liquid crystals," Phys. Rev. Lett. **86**, 2285–2288 (2001).
9. L. Oroszi, P. Galajda, H. Kirei, S. Bottka, and P. Ormos, "Direct measurement of torque in an optical trap and its application to double-strand DNA," Phys. Rev. Lett. **97**, 058301 (2006).
10. L. X. Chen, G. L. Zheng, J. Xu, B. Z. Zhang, and W. L. She, "Electrically controlled transfer of spin angular momentum of light in an optically active medium," Opt. Lett. **31**, 3474–3476 (2006).
11. L. X. Chen, G. L. Zheng, and W. L. She, "Electrically and magnetically controlled optical spanner based on the transfer of spin angular momentum of light in an optically active medium," Phys. Rev. A **75**, 061403 (2007).
12. G. Zheng and W. She, "Fast and wide-range continuously tunable Saolc-type filter based on periodically poled LiNbO3," Appl. Phys. B. **88**, 545–549 (2007).
13. K. Liu, and X. F. Chen, "Evolution of the optical polarization in a periodically poled superlattice with an external electric field," Phys. Rev. A **80**, 063808 (2009).
14. Y. Q. Lu, Z. L. Wan, Q. Wang, Y. X. Xi, and N. B. Ming, "Electro-optic effect of periodically poled optical superlattice LiNbO3 and its applications," Appl. Phys. Lett. **77**, 3719–3721 (2000).
15. S. Zhu, Y. Y. Zhu, and N. B. Ming, "Quasi-phase-matched third-harmonic generation in a quasi-periodic optical superlattice," Science **278**, 843–846 (1997).
16. G. L. Zheng, H. C. Wang, and W. L. She, "Wave coupling theory of quasi-phase-matched linear electro-optic effect," Opt. Express **14**, 5535–5540 (2006).
17. J. F. Nye, *Physical Properties of Crystals: Their Representation by Tensors and Matrices* (Oxford Univ. Press, 1985).
18. K. Liu, J. H. Shi, and X. F. Chen, "Linear polarization-state generator with high precision in periodically poled lithium niobate," Appl. Phys. Lett. **94**, 101106 (2009).
19. T. J. Wang and J. S. Chung, "Wavelength-tunable polarization converter utilizing the strain induced by proton exchange in lithium niobate," Appl. Phys. B **80**, 193–198 (2004).
20. L. Chen, J. Shi, X. Chen, and Y. Xia, "Photovoltaic effect in a periodically poled lithium niobate Solc-type wavelength filter," Appl. Phys. Lett. **88**, 121118 (2006).
21. Y. Q. Zhao, J. S. Edgar, G. D. M. Jeffries, D. McGloin, and D. T. Chiu, "Spin-to-orbital angular momentum conversion in a strongly focused optical beam," Phys. Rev. Lett. **99**, 073901 (2007).