

# Large phase shift via polarization-coupling cascading

Juan Huo and Xianfeng Chen\*

Department of Physics, the State Key Laboratory of Advanced Optical Communication Systems and Networks  
Shanghai Jiao Tong University, Shanghai 200240, China  
\*xfchen@sjtu.edu.cn

**Abstract:** Herein, we propose a phenomenon of “polarization-coupling (PC) cascading” generated in MgO doped periodically poled lithium niobate crystal (PPMgLN). PC cascading contributes to the effective electro-optical (EO) Kerr effect that is several orders of magnitude stronger than the classical ones. Experiment of Newton’s rings demonstrates the large phase accumulation during the PC cascaded processes, and the experimental data is identical with the theoretical simulation.

©2012 Optical Society of America

**OCIS codes:** (160.2100) Electro-optical materials; (160.4330) Nonlinear optical materials; (260.5430) Polarization; (260.1180) Crystal optics; (160.3730) Lithium niobate.

---

## References

1. J. M. R. Thomas and J. P. E. Taran, “Pulse distortions in mismatched second harmonic generation,” *Opt. Commun.* **4**(5), 329–334 (1972).
  2. G. I. Stegeman, D. J. Hagan, and L. Torner, “ $\chi^2$  cascading phenomena and their applications to all-optical signal processing, mode-locking, pulse compression and solitons,” *Opt. Quantum Electron.* **28**(12), 1691–1740 (1996).
  3. R. DeSalvo, D. J. Hagan, M. Sheik-Bahae, G. Stegeman, E. W. Van Stryland, and H. Vanherzeele, “Self-focusing and self-defocusing by cascaded second-order effects in KTP,” *Opt. Lett.* **17**(1), 28–30 (1992).
  4. W. E. Torruellas, Z. Wang, D. J. Hagan, E. W. VanStryland, G. I. Stegeman, L. Torner, and C. R. Menyuk, “Observation of two-dimensional spatial solitary waves in a quadratic medium,” *Phys. Rev. Lett.* **74**(25), 5036–5039 (1995).
  5. L. Misoguti, S. Backus, C. G. Durfee, R. Bartels, M. M. Murnane, and H. C. Kapteyn, “Generation of broadband VUV light using third-order cascaded processes,” *Phys. Rev. Lett.* **87**(1), 013601–013604 (2001).
  6. A. Fratolocci, R. Asquini, and G. Assanto, “Integrated electro-optic switch in liquid crystals,” *Opt. Express* **13**(1), 32–37 (2005).
  7. J. Zhang, J. S. Nelson, and Z. Chen, “Removal of a mirror image and enhancement of the signal-to-noise ratio in Fourier-domain optical coherence tomography using an electro-optic phase modulator,” *Opt. Lett.* **30**(2), 147–149 (2005).
  8. Y. Q. Lu, Z. L. Wan, Q. Wang, Y. X. Xi, and N. B. Ming, “Electro-optic effect of periodically poled optical superlattice LiNbO<sub>3</sub> and its applications,” *Appl. Phys. Lett.* **77**(23), 3719–3721 (2000).
  9. A. Yariv and P. Yeh, *Optical Waves in Crystals: Propagation and Control of Laser Radiation* (John Wiley & Sons, Inc., 1984).
  10. X. Chen, J. Shi, Y. Chen, Y. Zhu, Y. Xia, and Y. Chen, “Electro-optic Solc-type wavelength filter in periodically poled lithium niobate,” *Opt. Lett.* **28**(21), 2115–2117 (2003).
  11. J. Huo, K. Liu, and X. Chen, “1 x 2 precise electro-optic switch in periodically poled lithium niobate,” *Opt. Express* **18**(15), 15603–15608 (2010).
  12. K. Liu, J. Shi, and X. Chen, “Linear polarization-state generator with high precision in periodically poled lithium niobate,” *Appl. Phys. Lett.* **94**(10), 101106 (2009).
  13. Y. H. Chen and Y. C. Huang, “Actively Q-switched Nd:YVO<sub>4</sub> laser using an electro-optic periodically poled lithium niobate crystal as a laser Q-switch,” *Opt. Lett.* **28**(16), 1460–1462 (2003).
  14. S. Zhu, Y. Zhu, and N. Ming, “Quasi-phase-matched third-harmonic generation in a quasi-periodic optical superlattice,” *Science* **278**(5339), 843–846 (1997).
  15. G. Alexakis, N. Theofanous, A. Arapoyianni, M. Aillerie, C. Carabatos-Nedelec, and M. Fontana, “Measurement of quadratic electrooptic coefficients in LiNbO<sub>3</sub> using a variation of the FDEOM method,” *Opt. Quantum Electron.* **26**(12), 1043–1059 (1994).
  16. Y. Lee, N. Yu, C. S. Kee, D. K. Ko, Y. C. Noh, B. A. Yu, W. Shin, T. J. Eom, K. Oh, and J. Lee, “All-optical wavelength tuning in Solc filter based on Ti: PPLN waveguide,” *Electron. Lett.* **44**(1), 30–32 (2008).
-

## 1. Introduction

The history of optical cascading can be traced back to the 1970's, when J. M. Thomas et al. observed a cascade interaction between lights [1]. The importance of cascading, however, was not fully appreciated until 1996, when G. I. Stegeman studied cascading systematically and considered it as a promising direction to explore optical phenomena [2]. Consequently, in the following years, researchers have shown enormous interest in the possible exploration of various nonlinear effects such as nonlinear phase shifts, slow light, mode-locking, pulse compression, all-optical transistor action, and spatial solitons [3–5].

In this letter, we propose a new phenomenon named “polarization-coupling (PC) cascading”. PC cascading is modeled after the second harmonic generation (SHG) cascading [2] within MgO doped periodically poled lithium niobate crystal (PPMgLN). Based on this PC cascading effect, the effective nonlinearity of PPMgLN is determined by the transversely applied electric field, which is similar to the electro-optical (EO) Kerr effect, thus PC cascading effect could be defined as effective EO Kerr effect. Interestingly, the Kerr constant of this effective EO effect is several orders of magnitude larger than that in the classic counterparts, and could lead to large nonlinear phase shifts. The enhancement of the effective EO Kerr effect is proved by theoretical calculation, and the large phase shifts are demonstrated by experimental results. It has been shown that large phase shifts have extensive applications [6,7]. Therefore, the subject of controlling the phase shifts by electro-optical effect is widely studied and very important from both scientific and technological points of view.

## 2. Theoretical model and simulation

As is known, second harmonic generation (SHG) cascading occurs via up-conversion ( $\omega + \omega \rightarrow 2\omega$ ) followed by down-conversion ( $2\omega - \omega \rightarrow \omega$ ) [2]. The physical picture of the classic SHG cascading is shown in Fig. 1(a). Due to the phase velocity of the new fundamental frequency photon is inconsistent with the input one, the nonlinear phase shift is yielded via cascaded processes.

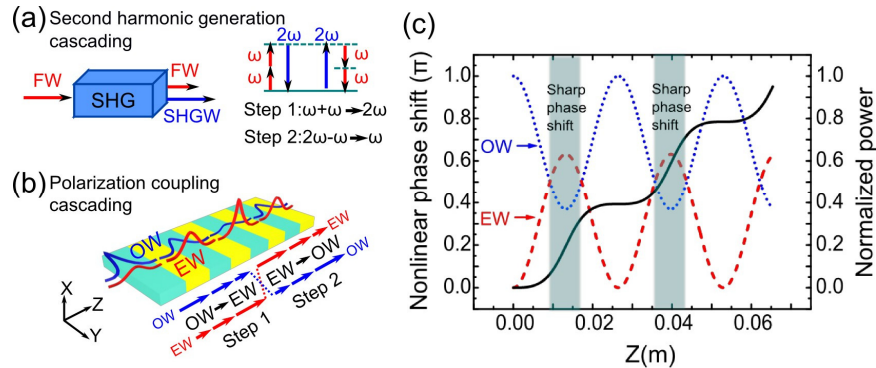


Fig. 1. The physic pictures of SHG cascading processes and PC cascading processes. (a), (b) The physical illustrations of the SHG cascading and PC cascaded processes. (c) The phase shifts inside PPMgLN, the external electric field is  $0.245 \text{ V} / \mu\text{m}$ .

By following the example of SHG cascading, we propose a new cascaded phenomenon—PC cascading in PPMgLN under non quasi-phase matching (NQPM) condition. These cascaded processes are divided into two steps: Taking ordinary wave (OW) incidence for example, the energy of OW flows to extraordinary wave (EW), but does not cause complete depletion of OW; then, the energy flows back from EW into OW, after approximately one coherence length. Because the regenerated OW is no longer in phase with the non-converted, a net OW phase is yielded (shown in Fig. 1(b)). The vectors in Fig. 1(b) show the energy

variation of OW and EW during the coupling processes. From Fig. 1(c), the increment of the nonlinear phase primarily occurs during the cycle wherein energy strongly exchanges between the two beams.

Here, PC cascading in a PPMgLN is considered when a transverse external electric field is applied along the PPMgLN. Based on the electro-optical effect, the optical axis of each domain are alternately aligned at the angles of  $+\theta$  and  $-\theta$ , with respect to the plane of polarization of the input light [8]. Energy coupling between OW and EW happens in these folded domains. The relative azimuth angle between the dielectric axes of two adjacent domains is very small so that the periodic alternation of the azimuth can be considered as a periodic small perturbation. In this case, the coupled-mode equations of the ordinary and extraordinary waves are [9]:

$$\begin{cases} dA_1 / dz = -i\kappa A_2 e^{i\Delta\beta z} \\ dA_2 / dz = -i\kappa A_1 e^{-i\Delta\beta z} \end{cases}, \quad (1)$$

with  $\Delta\beta = k_1 - k_2 - G_m$ ,  $G_m = 2\pi m / \Lambda$  and  $\kappa = -\frac{\omega n_o^2 n_e^2 \gamma_{51} E_y}{2c \sqrt{n_o n_e}} \frac{i(1 - \cos m\pi)}{m\pi}$ , ( $m = 1, 3, 5, 7, \dots$ ),

where  $A_1$  and  $A_2$  are the normalized complex amplitudes of OW and EW, respectively.  $\Delta\beta$  is the vector-mismatch;  $k_1$  and  $k_2$  are the corresponding wave vectors;  $G_m$  is the  $m^{\text{th}}$  reciprocal vector corresponding to the periodicity of poling;  $\Lambda$  is the period of the PPMgLN,  $\gamma_{51}$  is the electro-optical coefficient (its value is referred to [8]),  $E_y$  is the transverse electric field intensity,  $n_o$  and  $n_e$  are the refractive indices for the ordinary and extraordinary waves respectively, and with the initial condition (Assuming that the incident beam is OW):

$$\begin{cases} A_1(0) = 1 \\ A_2(0) = 0 \end{cases}. \quad (2)$$

The solution of Eq. (1) is given by

$$\begin{cases} A_1(z) = e^{i(\Delta\beta/2)z} [\cos sz - i \frac{\Delta\beta}{2s} \sin sz] \\ A_2(z) = e^{-i(\Delta\beta/2)z} (-i\kappa^*) \frac{\sin sz}{s} \end{cases}, \quad (3)$$

with  $s^2 = \kappa\kappa^* + (\Delta\beta/2)^2$ . For perfect quasi-phase matching (QPM) ( $\Delta\beta = 0$ ), the solution of Eq. (1) is simplified to  $A_1(z) = \cos(|\kappa|z)$  and  $A_2(z) = \sin(|\kappa|z)$ . This well-known QPM condition has opened the way to a host of optical devices such as narrowband Solc-type filters, electro-optical switches, precise polarization controllers and laser-Q switches [10–14]. However, the cascading phenomenon of phase-mismatch has not yet been understood. Thus, this study concentrates on the non-QPM solution ( $\Delta\beta \neq 0$ ), by studying the phase of OW and the EW, in which a rich variety of cascaded phenomena can occur.

From Eq. (3), we have  $A_2 = C \exp[-i(\Delta\beta/2)z]$ , with  $C = (-i\kappa^* / s) \sin(sz)$ .  $A_2$  is a real number. The phase of EW is easily obtained:

$$\Delta\Phi_e^{NL} = \begin{cases} -\frac{\Delta\beta}{2} z, C > 0 \\ -\frac{\Delta\beta}{2} z \pm \pi, C < 0 \end{cases}, \quad (4)$$

with  $\Delta\Phi_e^{NL}$  confined to interval  $[-\pi, \pi]$ . Eq. (4) suggests that  $\Delta\Phi_e^{NL}$  of EW varies linearly with phase-mismatch, and the “half-wave loss” happens. The phase of OW is more complicated. Thus, obtaining the approximate solution for large phase mismatch or low external electric field was attempted. Assume  $|A_1| \cong 1$ , and hence  $A_1(z) = \exp[-i\Delta\Phi_o^{NL}(z)]$ . From Eq. (3), the nonlinear phase shift of OW satisfies the following equation:

$$\Delta\Phi_o^{NL}(z=L) = -\frac{\Delta\beta L}{2} (1 - \sqrt{1 + (2\kappa^* \kappa / \Delta\beta)^2}). \quad (5)$$

For large phase-mismatch or low external electric field, we have  $|\Delta\beta| \gg |\kappa|$ . This nonlinear phase shift varies in proportion to the square of the electric field  $E_y$ . The variation is similar to the classic electro-optical Kerr effect and can be shown as:

$$\Delta\Phi_o^{NL} \cong \frac{|\kappa L|^2}{\Delta\beta L}. \quad (6)$$

Following the example of changing in refractive index of EO Kerr effect, an “effective” EO nonlinear refractive index by PC cascading can be introduced by:

$$\Delta n_o^{eff} = \frac{\omega n_e^3 n_o^2 \gamma_{51}^2}{\pi^2 c \Delta\beta} E_y^2. \quad (7)$$

In realm of nonlinear optics, the polarizability of a medium modified by optical fields can be given by  $P = P_0 + \chi_1 E + \chi_2 EE + \chi_3^{eff} EE + \chi_3 EEE \dots$ , where  $\chi_3^{eff} EE$  corresponds to the effective third order nonlinearity which is induced by second order nonlinearity  $\chi_2$  in SHG cascaded processes [2]. Similarly, during PC cascading the index of refraction modified by cascaded electro-optical effect can be given by:

$$n = n_0 + \frac{1}{2} \gamma n_0^3 E + \frac{1}{2} s^{eff} n_0^3 E^2 + \frac{1}{2} s n_0^3 E^2 + \dots \quad (8)$$

Just like  $\chi_2$  inducing an  $\chi_3^{eff}$  in SHG cascading,  $\gamma$  leads to an  $s^{eff}$  in PC cascading. Here  $s^{eff}$  is referred as the effective EO Kerr constant which could be enhanced during the effective electro-optical Kerr effect. Since in a PPMgLN the classical transverse electro-optical effect can only rotate the optical axis of the lithium niobate, the second term on the right side of Eq. (8) does not contribute to the nonlinear phase shift, and the actual second order Kerr effect is too weak to cause observable phenomenon and is negligible. In all, the observed large phase shift directly related to the effective EO Kerr effect, as indicated by the third term.

Supposing  $\Delta\beta = 1\pi/m$ ,  $\lambda = 600nm$ ,  $n_e = 2.1930$ ,  $n_o = 2.2829$  and  $\gamma_{51} = 32.6 pm/V$ , the effective Kerr constant  $s^{eff}$  will be  $1.04 \times 10^{-14} m^2/V^2$ , which is three orders of magnitude larger than the classical EO Kerr constant of lithium niobate,  $3.39 \times 10^{-17} m^2/V^2$  [15]. It should be noted that the enhanced EO Kerr constant is an effective effect which has nothing to do with the particular material but mainly governed by the periodical index modulation.

The precise phase-variations with external electric field of EW and OW governed by Eq. (3) are shown in Fig. 2 which indicates the potential of nonlinear EO phase modulators. The four solid lines present the variation of the nonlinear phase shifts with external electric field at different vector-mismatches,  $\Delta\beta = 121\pi/m, 16\pi/m, -16\pi/m$  and  $-121\pi/m$ , while dash lines present the transmission with electric fields, showing the energy coupling between OW and EW in PPMgLN. By keeping an appropriate nonzero value of the phase-mismatch, the transmission of EW and OW vary from 0 to 0.99 and 0.01 to 1, respectively. Just by adjusting the vector-mismatch, the magnitude and sign of the effective nonlinearity, i.e. nonlinear phase can be varied. As shown in Fig. 2(a), the large phase shifts only occur when the EW

experiences a “half-wave loss”, which means the phase maintains constant except at some critical electric fields. While in Fig. 2(b), in the vicinity of some critical electric fields, tiny changes of the electric field can cause large changes of phase for OW. It should be noted that the conventional phase modulators based on Pockels effect only realize linear phase shift and are unable to obtain sharp phase shift or half-wave loss.

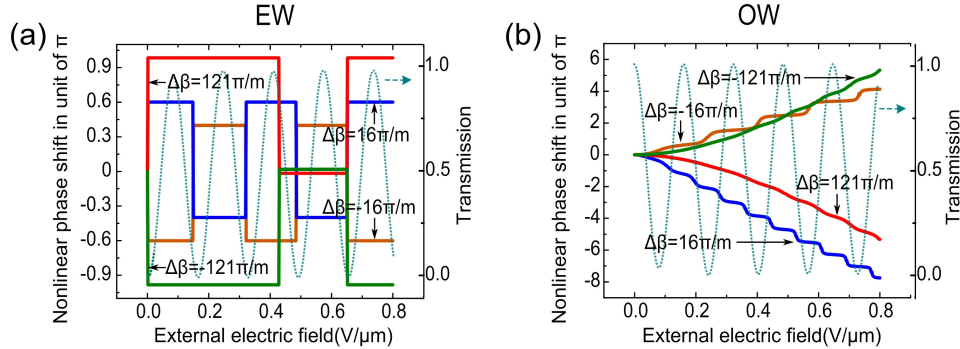


Fig. 2. Typical variation of the nonlinear phase shifts with external electric field for EW and OW. The dashed lines present the transmission as a function of electric fields for  $T = 21^\circ\text{C}$ . The length of PPMgLN for calculation was set as 50 mm.

### 3. Experiment and results

The schematic of the experimental setup is shown in Fig. 3. A scheme of March-Zehnder interference was utilized to investigate the phase shifts. The wavelength of the He-Ne laser is 632.8nm, which almost satisfies the third order phase-matching condition with poling period of PPMgLN to be 21.1  $\mu\text{m}$ . The laser power is 8 milliwatts. First, the horizontally polarized incident beam was separated by a beam splitter (BS) with one beam passing through PPMgLN and the other in free space. Thus, the incident beam in PPMgLN is OW.

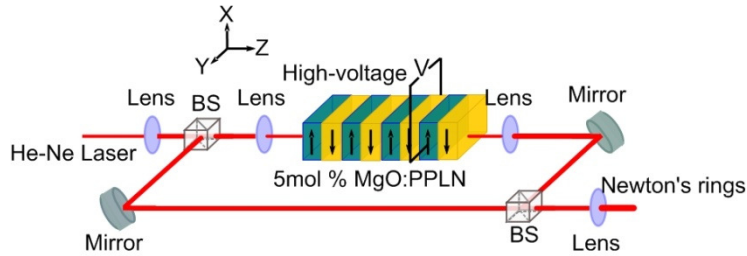


Fig. 3. Experimental setup for demonstrating the nonlinear phase shifts yielded in PC cascading. A PPMgLN crystal, which is 5mol%MgO:PPLN, with the period of 21.1  $\mu\text{m}$  and the length of 50 mm. High-voltage is used to supply transverse electric fields.

The experimental results are shown in Fig. 4(a) and (b). By varying the transverse electric field from 0 to 0.56  $\text{V}/\mu\text{m}$ , we found that the interference fringes’ “light-dark” changed at different temperatures (different vector-mismatches). When the vector-mismatch was around  $-1000 \pi / m$  ( $23^\circ\text{C}$ , Fig. 4(a)), the Newton’s rings had hardly any change in varying electric field. It concludes that the nonlinear phase shift cannot be accumulated to  $\pi$  at very large vector-mismatch. When the vector-mismatch was changed to approximate  $-150 \pi / m$  by lowering temperature to  $21.3^\circ\text{C}$  (Fig. 4(b)), the interference fringes experienced three times “light-dark” alternation at some certain electric fields, 0.28  $\text{V}/\mu\text{m}$ , 0.44  $\text{V}/\mu\text{m}$  and 0.56  $\text{V}/\mu\text{m}$ . Each change means a  $\pi$  phase shift. By this token, transverse electric field is not the determinant of the large phase shifts. In a PPMgLN, the classical transverse electro-optical

effect is only able to rotate the optical axis of the lithium niobate, which indicates the large phase shift cannot be the classic EO Pockels effect. Figure 4(c) shows the comparison of the simulated curve and the experimental results at the temperature of  $21.3^{\circ}\text{C}$ . The critical electric fields leading to  $\pi$  phase shifts with the vector-mismatch of approximate  $-150\pi/m$  (in Fig. 4(b)) are shown as the color dots in Fig. 4(c). Thus, we can see that the experimental data well fit the theoretical curve. Because of some unavoidable errors, for instance, the refractive indices of the PPMgLN sample we used are not consistent with the simulated values; the experimental data can't be in full agreement with the theoretical ones. It should be noted that the PPLN waveguide has been successfully proposed recently [16], where the gap between the electrodes can be as short as  $10\ \mu\text{m}$ , so that only several Volts is enough to generate the large phase shifts.

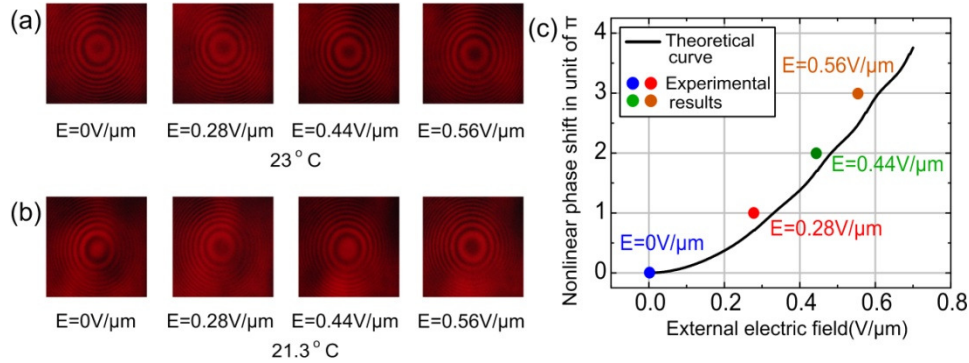


Fig. 4. The comparison of experimental results and theoretical simulation for demonstrating the enhanced phase shift yielded in PC cascading; (a), (b) the center experiences “dark-light” changes by changing the transverse electric field at different vector-mismatches. (c), the comparison of simulated curve and experimental results at  $21.3^{\circ}\text{C}$ .

#### 4. Conclusion

In summary, PC cascading was demonstrated in PPMgLN through experiment and theory. Nonlinear phase shifts generated from the cascaded processes between a pair of orthogonal beams i.e. OW and EW have been investigated. The results provide a method which can be used to achieve enhanced EO Kerr effect. It should be noted that the PC cascading proposed here is different from the SHG cascading phenomenon, because the former belongs to linear optics, and the latter is classified to nonlinear optics. With a different physical understanding, PC cascading may trigger interest in a wide range of fields.

#### Acknowledgments

This research was supported by the National Natural Science Foundation of China (Grant No. 61125503, 61078009), the National Basic Research Program “973” of China (Grant No. 2011CB808101), the Foundation for Development of Science and Technology of Shanghai (Grant No. 11XD1402600) and the Open Fund of the State Key Laboratory of High Field Laser Physics.