Hidden equivalence in the collective emission from a dilute atomic cloud

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We investigate the collective spontaneous emission from an ensemble of two-level atoms evenly distributed inside a sphere with a low density. An initial symmetric single-excitation state is considered and polarizations for all atoms are assumed to be aligned in the same direction. We find that the superradiant decay rate of the ensemble exhibits oscillatory features dependent on the radius of the sphere and the atomic radiation wavelength, indicating that the collective emission rate can be less than the single-atom decay rate for certain parameters. Moreover, the system exhibits a hidden equivalence where the collective emission from the ensemble gives the same radiation rate as the case of a two-atom model where one atom is at the center of the sphere and the other effective large atom, composed of all other atoms, is localized at the edge of the sphere along the polarization direction. Our result hence provides a potential method towards exploring complex many-body physics by simplifying the model into a two-body problem.

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I. INTRODUCTION

The collective emission from an atomic ensemble is of great importance in atomic physics and quantum optics [1]. It has been shown that a cloud of identical atoms is synchronized to emit coherently. In particular, for a collection of N atoms with the size of the ensemble being much smaller than the atomic radiation wavelength λ, the collective emission rate from the symmetric single-excitation Dicke state is N times larger than a single-atom spontaneous emission rate Γ^{11} [1]. Due to the difficulty in preparing small atomic ensembles with a high density, experimental studies of superradiance have been done with large-size ensembles [2,3], where it is assumed that Dicke states can be found in the course of temporal evolution of a fully excited system [3–5]. Moreover, Dicke states can also be generated using projective measurements with low success probabilities [6–8].

Single-photon collective spontaneous emission has drawn enormous interest in the past because of its potential applications in quantum information processing such as producing coherent emission without coherent pumping [1], realizing single-photon sources [9,10], and generating quantum states for quantum memory and quantum networking [11,12]. The modification of the spontaneous emission of one atom in the presence of the other $N-1$ atoms in a cavity has been investigated in Refs. [13–16]. The dynamics of the system composed of N two-level atoms with one atom excited initially in free space has been studied, which exhibits a radiation suppression phenomenon [17]. In the past decade, the timed Dicke state [18–28] where the ensemble is excited by a single photon has been intensively explored. The decay rate of such a state is approximately proportional to $N\Gamma^{11}\lambda^2/R^2$ [19–21,27], where R is the radius of the sample. Potential applications of the single-photon superradiant state include quantum metrology [24], quantum simulation of topological physics [26], and quantum control of spontaneous emission and ultrafast readout [27]. Related but different from the above works, in Ref. [29] the behavior of the collective spontaneous emission rate in the vector model was explored in the case of weak excitation. Nevertheless, the polarization effect in the collective emission due to the vector nature of the electromagnetic field has not been explored in great detail for the large-size single-photon excited atomic ensemble.

In this paper we investigate the collective spontaneous emission from an equally distributed spherical cloud of N atoms, where an initial symmetric single-excitation Dicke state is considered. Taking into account the effect of polarization, we study the superradiant decay rate of such an ensemble with a low density and find that the collective emission rate also depends on the atomic number N, the radius in the unit of wavelength R/λ, and the single-atom spontaneous emission rate Γ^{11}. However, the collective emission rate exhibits an oscillatory behavior on R in a period of λ. Such an oscillatory feature makes it possible for the superradiant decay rate for the atomic ensemble to become less than the single-atom decay rate, which corresponds to the suppression of the emission. We also find a hidden equivalence for the modulations of the one-atom spontaneous emission rate, i.e., our model is mathematically equal to the case that one atom sits at the center and all other atoms are localized at the edge of the sphere, forming an effective large atom. The many-body problem is thus transformed into an effective two-body problem.

II. THEORETICAL ANALYSIS

We start with a model composed of N two-level atoms coupled to a radiation field. The Hamiltonian that describes the atoms and radiation field can be written as $H_0=\sum_l^N \omega_0\sigma_l^+\sigma_l^- + \sum_{\mathbf{k}\lambda}\omega_{\mathbf{k}\lambda}a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda}$. Here ω_0 is the atomic transition frequency, $\sigma_l^+=|e_l\rangle\langle g_l|$ and $\sigma_l^-=|g_l\rangle\langle e_l|$ are the

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raising and lowering operators of the lth atom, respectively, and $a^\dagger_{\mathbf{k}\lambda}$ and $a_{\mathbf{k}\lambda}$ are, respectively, the creation and annihilation operators for a photon with the momentum $\mathbf{k}$, the frequency $\omega_{\mathbf{k}}$, and the polarization λ. The atom-field interaction is described by $H_I = -\sum_{l=1}^{N}(\boldsymbol{\mu}^l\sigma_l^+ + \boldsymbol{\mu}^{l*}\sigma_l^-)\cdot\mathbf{E}(\mathbf{r}_l)$ under the dipole approximation, where $\boldsymbol{\mu}^l$ is the electric dipole transition matrix element and $\mathbf{E}(\mathbf{r}_l)$ denotes the radiation field operator at the position $\mathbf{r}_l$ for the lth atom. The Hamiltonian can be rewritten in the interaction picture as $H_I(t) = -\sum_{l=1}^{N}(\boldsymbol{\mu}^l\sigma_l^+e^{i\omega_0 t} + \boldsymbol{\mu}^{l*}\sigma_l^-e^{-i\omega_0 t})\cdot\mathbf{E}(\mathbf{r}_l,t)$, where $\mathbf{E}(\mathbf{r}_l,t) = e^{iH_0 t}\mathbf{E}(\mathbf{r}_l)e^{-iH_0 t}$. We decompose the electric-field operator into positive- and negative-frequency components $\mathbf{E}(\mathbf{r}_l,t) = \mathbf{E}^+(\mathbf{r}_l,t) + \mathbf{E}^-(\mathbf{r}_l,t)$, where $\mathbf{E}^+(\mathbf{r}_l,t)|0\rangle = 0$ and $\langle 0|\mathbf{E}^-(\mathbf{r}_l,t) = 0$ [30]. By applying the rotating-wave approximation, we obtain

$$H_I(t) = -\sum_{l=1}^{N}\left[\mu_i^l E_i^+(\mathbf{r}_l,t)\sigma_l^+e^{i\omega_0 t} + \mu_i^{l*}E_i^-(\mathbf{r}_l,t)\sigma_l^-e^{-i\omega_0 t}\right]. \tag{1}$$

Here the Einstein summation convention is applied to the polarization i, where the index i is x, y, and z in the Cartesian coordinate system.

We consider an equally distributed N-atom spherical ensemble and assume that initially there is one atom excited and the field is under the vacuum state, which corresponds to a symmetric single-excitation Dicke state: $|\varphi(0)\rangle = \frac{1}{\sqrt{N}}\sum_{l=1}^{N}|g_1g_2\cdots e_l\cdots g_N\rangle|0\rangle$. At the time t, the state vector becomes [31,32]

$$\begin{aligned}|\varphi(t)\rangle &= \sum_{l=1}^{N} b_l(t)|g_1g_2\cdots e_l\cdots g_N\rangle|0\rangle\\ &\quad + \sum_{\mathbf{k}\lambda} d_{\mathbf{k}\lambda}(t)|g_1g_2\cdots g_N\rangle|1_{\mathbf{k}\lambda}\rangle,\end{aligned} \tag{2}$$

where $b_l(0) = 1/\sqrt{N}$ and $|1_{\mathbf{k}\lambda}\rangle$ denotes a photon in the mode $(\mathbf{k},\lambda)$. Here we restrict our analysis to the single-photon limit.

The dynamical evolution of the state vector is determined by Schrödinger's equation. In the weak-interaction limit, one can apply the Markovian approximation [33] and derive the evolution equation for the state probability amplitudes in Eq. (2) as

$$\dot{b}_l(t) = -\sum_{m=1}^{N} L^{lm} b_m(t), \tag{3}$$

where $L^{lm} = \mu_i^l\mu_j^{m*}\int_0^\infty du\, e^{i\omega_0 u}G_{ij}^{lm}(u)$, $u = t - t'$, and $G_{ij}^{lm}(u) = \langle 0|E_i^+(\mathbf{r}_l,t)E_j^-(\mathbf{r}_m,t')|0\rangle$ is the field correlation function. Here L^{lm} can be rewritten as

$$L^{lm} = \tfrac{1}{2}\mu_i^l\mu_j^{m*}\mathcal{G}_{ij}^{lm}(\omega_0) + i\mu_i^l\mu_j^{m*}\mathcal{K}_{ij}^{lm}(\omega_0), \tag{4}$$

with

$$\begin{aligned}\mathcal{G}_{ij}^{lm}(\omega_0) &= \int_{-\infty}^{\infty} du\, e^{i\omega_0 u}G_{ij}^{lm}(u),\\ \mathcal{K}_{ij}^{lm}(\omega_0) &= -\frac{\mathrm{P}}{2\pi}\int_{-\infty}^{\infty}\frac{\mathcal{G}_{ij}^{lm}(\omega)}{\omega-\omega_0}d\omega.\end{aligned} \tag{5}$$

Here $\mu_i^l\mu_j^{l*}\mathcal{G}_{ij}^{ll}(\omega_0) \equiv \Gamma^{ll}$ is the spontaneous emission rate for the lth atom [34]; $\mu_i^l\mu_j^{m*}\mathcal{G}_{ij}^{lm}(\omega_0) \equiv \Gamma^{lm}$, with $l \neq m$, defines the modulation of the spontaneous emission rate of the lth atom due to the presence of the mth atom [35]; $\mu_i^l\mu_j^{l*}\mathcal{K}_{ij}^{ll}(\omega_0)$ gives the energy shift of the lth atom; $\mu_i^l\mu_j^{m*}\mathcal{K}_{ij}^{lm}(\omega_0) \equiv V^{lm}$, with $l \neq m$, defines the dipole-dipole interaction potential between the lth and the mth atoms; and P denotes the Cauchy principal value.

With a large ensemble, we assume that the evolution dynamics of each atom is approximately the same, i.e., $b_m(t) = b_l(t) \equiv b(t)$. This assumption is true when the size of the sphere goes to infinity. For a dilute large atomic cloud, we can still take this approximation: For atoms away from the boundary of the sphere, there is almost no difference in the atom-field interaction between the atoms, so the excitation probability is nearly the same; for atoms near the boundary of the sphere, their contribution is negligible for a large ensemble. Under this assumption, we have

$$b(t) = \frac{1}{\sqrt{N}}\exp\left[-\sum_{m=1}^{N}\left(\frac{1}{2}\Gamma^{1m} + iV^{1m}\right)t\right], \tag{6}$$

where we set the first atom as the atom at the center of the spherical atomic cloud. We clarify here that the first atom is not necessarily the atom at the center, but can be any atom near the center of the sphere due to the similarity of the atom's excitations under our approximation.

We use the solution in Eq. (6) to investigate the collective decay rate of the atomic ensembles. The transition probability for an atomic system from the initial state $\rho(0)$ to the final state ρ_f can be expressed as $P(t) = \mathrm{Tr}[\rho_f\rho(t)]$ [36]. Here the time-dependent reduced density matrix $\rho(t) = \mathrm{Tr_f}[|\varphi(t)\rangle\langle\varphi(t)|]$ is obtained by tracing the density matrix of the coupled system over the field degrees of freedom. The final state ρ_f is the density matrix of the atomic ground state. As a result, one obtains $P(t) = 1 - N|b(t)|^2$. The collective emission rate Γ is defined as the transition probability per unit time at $t = 0$ [36]:

$$\Gamma = \partial_t P(t)|_{t=0} = \Gamma^{11} + \Gamma_M. \tag{7}$$

Here we take the transition matrix element $\boldsymbol{\mu}^m$ to be real, for simplicity. Equation (7) shows that the superradiant decay rate of the atomic ensemble is a summation between the spontaneous emission rate from one atom Γ^{11} and the corresponding modulations on the same atom due to the presence of other atoms

$$\Gamma_M = \sum_{m=2}^{N}\Gamma^{1m}. \tag{8}$$

Our next step is to calculate the modulations Γ_M to explore the collective decay rate Γ. We assume here that the polarizations of all atoms have the same directions and are aligned along the z axis as shown in Fig. 1(a). To calculate Γ_M in Eq. (8), we can write $\Gamma^{lm} = \Gamma_\parallel^{lm} + \Gamma_\perp^{lm}$ for the two arbitrary separated lth and mth atoms. Here $\Gamma_\parallel^{lm}$ ($\Gamma_\perp^{lm}$) is defined as the contribution due to the polarizations of atoms projected in (perpendicular to) the direction parallel to the line joining two atoms, with the corresponding transition matrix element

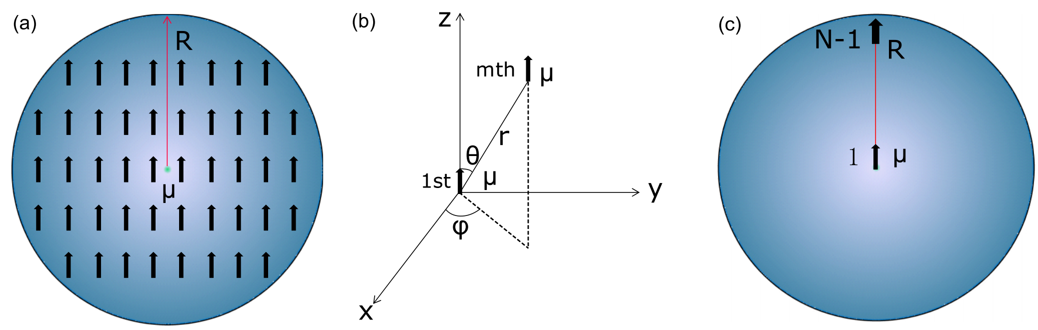

FIG. 1. (a) Equally distributed spherical cloud of atoms where their electric dipoles have the same magnitudes and orientations. (b) The spherical coordinate describes the relativity positions between the mth atom and the first atom located at the center of the sphere. The z axis is along the polarization direction. (c) Schematic illustration of an equivalent two-atom model in which one atom is at the center and another effective large atom with the dipole $(N-1)\mu$ is set at the edge of the sphere.

component $\mu_{\parallel}^{l/m}$ ($\mu_{\perp}^{l/m}$). Further, $\Gamma_{\parallel}^{lm}$ and $\Gamma_{\perp}^{lm}$ are obtained as [37]

$$\Gamma_{\parallel}^{lm} = \frac{\mu_{\parallel}^{l}\mu_{\parallel}^{m*}\omega_0^3}{\pi\varepsilon_0 c^3}\frac{\sin\eta - \eta\cos\eta}{\eta^3}, \tag{9}$$

$$\Gamma_{\perp}^{lm} = \frac{\mu_{\perp}^{l}\mu_{\perp}^{m*}\omega_0^3}{\pi\varepsilon_0 c^3}\frac{(\eta^2-1)\sin\eta + \eta\cos\eta}{2\eta^3}, \tag{10}$$

where $\eta \equiv r\omega_0/c$ is the phase difference between the radiation emitted by these two atoms and r is the separation distance between them. We further assume that electric dipoles of atoms have the same magnitudes $|\boldsymbol{\mu}^m| = \mu$, which gives $\mu_{\parallel}^1\mu_{\parallel}^{m*} = \mu^2\cos^2\theta$, where $\mu_{\perp}^1\mu_{\perp}^{m*} = \mu^2\sin^2\theta$ and θ is the polar angle of the mth atom [see Fig. 1(b)]. Under the condition $N \gg 1$, Γ_M defined in Eq. (8) becomes $\frac{N-1}{V}\int_0^R r^2 dr \int_0^{2\pi} d\varphi \int_0^{\pi} \sin(\theta)\,\Gamma^{1m} d\theta$, where V is the volume of the sphere. This leads to

$$\Gamma_M = 3(N-1)\Gamma^{11}\frac{\sin(\xi) - \xi\cos\xi}{\xi^3}, \tag{11}$$

where $\Gamma^{11} = \mu^2\omega_0^3/3\pi\varepsilon_0 c^3$ is the spontaneous decay rate of the atom and $\xi \equiv R\omega_0/c = 2\pi R/\lambda$ is the phase difference between the radiation emitted by the atom at the center and the ones at the edge. One can see that the collective emission rate of such a spherical atomic cloud depends on the particle number N, the phase difference ξ, and the decay rate of a single atom Γ^{11}.

One can find the similarity between Eqs. (9) and (11). From Eq. (9), if one sets the first atom at the center and $N-1$ atoms on the $+z$ axis at the edge of the sphere with their polarizations remaining along the z axis, as illustrated in Fig. 1(c), the modulations of decay rate of the first atom will be same as the case described by Eq. (11) in Fig. 1(a). Therefore, our system exhibits a hidden equivalence to the effective two-atom model in Fig. 1(c) with the electric dipole transition matrix element of the atom at the center being $\boldsymbol{\mu}$ and that of another effective large atom being $(N-1)\boldsymbol{\mu}$.

To see the properties of the collective decay rate in Eq. (7), we plot the function $f(\xi) \equiv 3[\sin(\xi) - \xi\cos\xi]/\xi^3$ from the modulations Γ_M in Eq. (11) as a function of the phase difference ξ in Fig. 2. One can see an oscillatory behavior dependent on the radius R. The period of the oscillation is equal to the resonant radiation wavelength λ, indicating the radius-dependent resonant feature of the sphere. Here $f(\xi) > 0$ corresponds to the enhanced collective decay rate of the atomic ensemble compared to the decay rate of a single atom, which exhibits a superradiant feature of the ensemble. As for $f(\xi) < 0$, it gives a decreased collective decay rate, which corresponds to the suppression of the emission. Therefore, with the radius of the sphere increases, one sees the transition between enhanced collective emission and the suppressed emission.

Figure 2 shows that the minimal value of $f(\xi)$ can be reached near the neighbor of $\xi = 2k\pi$, i.e., $R = k\lambda$, where $k \geqslant 1$ is an integer. At each minimum, the decay rate of the ensemble should be physically larger than zero. This gives a constraint on the system $1 \ll N \leqslant 4k^2\pi^2/3$, which can be written in another form $3/4\pi k^3 \ll n\lambda^3 \leqslant \pi/k$. Here $n = N/V$ is the atomic density. This constraint indicates that the average separation between atoms is comparable to or larger than the resonant radiation wavelength λ. Thus the atomic cloud must be dilute. In our model, the atoms are distinguishable. When the distance between atoms is too small, the current model is no longer valid. On the other hand, the constraint also provides a restriction on the atomic density by the radius in the unit of wavelength in our model.

In Fig. 3 we plot the collective decay rate versus the phase difference ξ and the atomic density n. From Fig. 3(a) one sees that the periodicity is only dependent on ξ. With a larger n, the contrast between the maximum and minimum values of Γ increases. To better study this plot, we consider two cases. (i) The number of atoms is constant, i.e., $N = 1000$.

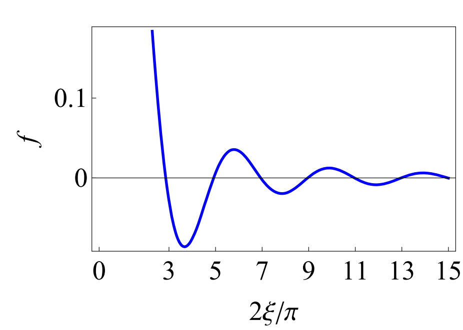

FIG. 2. Plot of $f(\xi) \equiv 3(\sin\xi - \xi\cos\xi)/\xi^3$ as a function of the phase difference ξ.

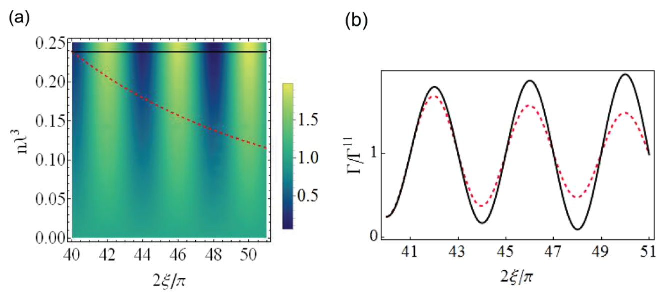

FIG. 3. (a) Collective spontaneous emission rate Γ/Γ^{11} as a function of the phase difference ξ and the atomic density n. The black line denotes the constant atomic density $n = 3/4\pi\lambda^3$ and the red dashed line the constant atomic number $N = 1000$. (b) Collective decay rate as a function of the phase difference ξ for constant atomic density $n = 3/4\pi\lambda^3$ (black solid line) and constant number of atoms $N = 1000$ (red dashed line).

The radius of the atomic cloud is taken in the range $R \geqslant 10\lambda$ to make it satisfy the constraint. (ii) The atomic density is constant, for instance, $n = 3/4\pi\lambda^3$. With this atomic density, for a radius $R = 10\lambda$, there are 1000 atoms in the sphere. As a comparison, we plot the evolution of the superradiant decay rate versus ξ for both cases in Fig. 3(b). The collective emission rate exhibits an oscillatory feature on the radius R with a period λ. With the increase of radius, the amplitude of the collective decay rate decreases successively for case (i), but increases gradually for case (ii). For case (i), the density of the atomic ensemble decreases when one increases the radius of the sphere and keeps the number of atoms constant, which causes the decrease of Γ. However, for case (ii), as one fixes the density of the atomic cloud and a bigger ξ gives a larger N, which results in an increase of Γ. Moreover, in case (ii), the radius R should not be greater than 13λ to meet the constraint for atomic density which is restricted by the radius in the unit of wavelength. Figure 3 characterizes the general properties of the superradiant emission rate versus the phase difference ξ for constant particle number and constant atomic density.

For an ensemble of identical atoms, the dipole orientations are usually chaotic and randomly distributed. When one applies an external resonant field to the ensemble, the transition dipole moment of each particle will be the same and the initial Dicke state can be found in the evolution of the excited system [3–5]. Therefore, our model is feasible in the experiment.

III. CONCLUSION

In summary, we have explored the collective emission from a dilute atomic cloud and found a hidden equivalence. A spherical ensemble of two-level atoms, which is prepared initially as a symmetric single-excitation state, has been studied. The polarizations for all atoms are considered to be aligned in the same direction. The vector nature of the electromagnetic field has been explored in great detail in our model. The results exhibit an oscillatory feature for the collective decay rate. We noticed that our model has a result equivalent to the emission from an effective two-atom model. Our work therefore points to a simplified route to explore complicated physics problems in N-atom collective coherent emission, which is important in the field of quantum information processing, and also holds promise for a better understanding of related applications such as coherence-enhanced sky lasing [38,39], superradiance in ionized gas [40,41], and quantum amplification by collective emission [42–44] in the future.

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